Homework 2

1) Q.2.4

The set of all wave functions must satisfy the boundary conditions as well as the conditions that allow us to interpret the square of the magnitude of the wavefunctions in terms of probability. The set of eigenfunctions must satisfy an additional condition given by the eigenvalue equation. Because some wavefunctions will not satisfy this equation, the set of wavefunctions is larger.

2) vector (1, 1) an eigenfunction of $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$? what about (1,-1)

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0\cdot1+1\cdot1 \\ 1\cdot1+0\cdot1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \text{yes, } \lambda = 1 \quad \checkmark$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0\cdot1+1\cdot-1 \\ 1\cdot1+0\cdot-1 \end{pmatrix} \Rightarrow \begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad \text{yes, } \lambda = -1$$

3) Normalize e^{-ax^2} over $(-\infty, +\infty)$

$$1 = N^2 \int_{-\infty}^{\infty} \left(e^{-ax^2}\right)^2 dx \Rightarrow N^2 \int_{-\infty}^{\infty} e^{-2ax^2} dx = 1$$

Using table of integrals: $\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$ where $a = +2a$

$$N^2\left(\sqrt{\frac{\pi}{2a}}\right) = 1$$

$$\sqrt{\frac{2a}{\pi}} = N^2 \qquad N = \left(\frac{2a}{\pi}\right)^{1/4}$$

Normalized $\Psi = \left(\frac{2a}{\pi}\right)^{1/4} e^{-ax^2}$

4) p 2.31

a) $\frac{x^2}{8}\frac{d^2}{dx^2}\left(x^2\right) \Rightarrow \frac{x^2}{8}\left(2\right) \Rightarrow \frac{1}{4}x^2$ yes, $\lambda = 1/4$

b) $\left(x^3\frac{\partial^3}{\partial x^3} + y^3\frac{\partial^3}{\partial y^3}\right)\left(x^3+y^3\right) \Rightarrow x^3\frac{\partial^3}{\partial x^3}(x^3+y^3) + y^3\frac{\partial^3}{\partial y^3}(x^3+y^3)$

$\Rightarrow x^3(6) + y^3(6) \Rightarrow 6(x^3+y^3)$ yes $\lambda = 6$

c) $\frac{\partial^4}{\partial\theta^4}\sin 2\theta\cos\phi \Rightarrow 16\sin 2\theta\cos\phi$ yes $\lambda = 16$

5) p 2.11 function $Ae^{-ik_1x}e^{-ik_2y}e^{-ik_3z}$

a) $\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\left(Ae^{-ik_1x}e^{-ik_2y}e^{-ik_3z}\right)$

$A\left[(-ik_1)e^{-ik_1x}e^{-ik_2y}e^{-ik_3z} + (-ik_2)e^{-ik_1x}e^{-ik_2y}e^{-ik_3z} + (-ik_3)e^{-ik_1x}e^{-ik_2y}e^{-ik_3z}\right]$

$-i\left(Ae^{-ik_1x}e^{-ik_2y}e^{-ik_3z}\right)(k_1+k_2+k_3)$

eigenvalue $= -i(k_1+k_2+k_3)$

b) $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\left(\text{function}\right)$

$A\left[((-i)^2k_1^2)(\text{function}) + ((-i)^2k_2^2)(\text{function}) + ((-i)^2k_3^2)(\text{function})\right]$

$-1(A)\left(e^{-ik_1x}e^{-ik_2y}e^{-ik_3z}\right)\left(k_1^2+k_2^2+k_3^2\right)$

eigenvalue $= -\left[k_1^2+k_2^2+k_3^2\right]$

P13.35 Express the following complex numbers in the form $a + ib$.

a. $2e^{3i\pi/2}$

b. $4\sqrt{3}\, e^{i\pi/4}$

c. $e^{i\pi}$

d. $\dfrac{\sqrt{5}}{1+\sqrt{2}} e^{i\pi/4}$

To convert to the form $a + ib$ from $Ae^{i\phi}$, we use the equations $\operatorname{Re} z = |z|\cos\theta$ and $\operatorname{Im} z = |z|\sin\theta$ and $|z| = A$.

(a) $$2e^{3i\pi/2} = 2\cos\frac{3\pi}{2} + \left(2\sin\frac{3\pi}{2}\right)i = -2i$$

(b) $$4\sqrt{3}e^{i\pi/4} = 4\sqrt{3}\cos\frac{\pi}{4} + \left(4\sqrt{3}\sin\frac{\pi}{4}\right)i = \frac{4\sqrt{3}}{\sqrt{2}} + \frac{4\sqrt{3}}{\sqrt{2}}i$$
$$= 2\sqrt{6} + 2\sqrt{6}i$$

(c) $$e^{i\pi} = \cos\pi + (\sin\pi)i = -1$$