Additional Notes Chapters 2 and 3

$\psi^*(x)\psi(x) \equiv$ probability of finding particle at *x*

$\int_{-\infty}^{\infty} \psi^* \psi \, dx = 1 \quad \longleftarrow$ probability of finding particle some where

(assumes ψ normalized)

ψ continuous, single valued ψ' continuous

Single measurement of an observable $\hat{A}$ can only give an eigenvalue of $\hat{A}$.
If ψ not an eigenfunction of *A*, one can still calculate an average

$$\langle \hat{A} \rangle = \frac{\int \psi H \psi \, d\tau}{\int \psi \psi \, d\tau}$$

here τ denotes the variable(s) being integrated over.

Consider the case where ϕ_1 and ϕ_2 are eigenfunctions of $\hat{A}$

$$\hat{A}\phi_1 = a_1\phi_1$$

$$\hat{A}\phi_2 = a_2\phi_2$$

and suppose you have the wavefunction

$$\psi = \frac{1}{2}\phi_1 + \frac{\sqrt{3}}{2}\phi_2$$

$$\int \psi^* \psi \, d\tau = \frac{1}{4}\int \left(\phi_1\phi_1 + \underbrace{2\sqrt{3}\phi_1\phi_2}_{\to\ 0} + 3\phi_2\phi_2 \right) d\tau$$

$$= \frac{1}{4}[1+3] = 1$$

So ψ is normalized

It has been assumed that ϕ_1 and ϕ_2 are real

$$\int \psi^* \hat{A} \psi \, d\tau$$

$$\frac{1}{4}\int \left(\phi_1 + \sqrt{3}\phi_2 \right) \hat{A} \left(\phi_1 + \sqrt{3}\phi_2 \right) d\tau$$

$$= \frac{1}{4}\int \left(\phi_1 + \sqrt{3}\phi_2 \right) \left[a_1\phi_1 + \sqrt{3}a_2\phi_2 \right] d\tau$$

$$= \frac{1}{4}\left[a_1 + 3a_2 \right]$$

¼ of measurements give a_1
¾ of measurements give a_2

Now consider the example from the text of three superposition states where ϕ_1, ϕ_2, ϕ_3 are eigenfunctions of an operator $\hat{A}$.

$$\psi_1(x) = \frac{\sqrt{11}}{4}\phi_1(x) + \frac{1}{4}\phi_2(x) + \frac{1}{2}\phi_3(x)$$

$$\psi_2(x) = \frac{1}{2}\phi_1(x) + \frac{1}{4}\phi_2(x) + \frac{\sqrt{11}}{4}\phi_3(x)$$

$$\psi_3(x) = \frac{1}{2}\phi_1(x) + \frac{\sqrt{11}}{4}\phi_2(x) + \frac{1}{4}\phi_3(x)$$

$$\left.\begin{array}{l} \phi_1 \rightarrow a_1 \\ \phi_2 \rightarrow 4a_1 \\ \phi_3 \rightarrow 9a_1 \end{array}\right\} \text{eigenvalues}$$

All measurements give one of these three values. But probabilities differ, depending on whether one has ψ_1, ψ_2, or ψ_3.

To determine average, need to do measurements on many identical systems.

Cannot get the answer by doing multiple measurements on one system. Why?

Suppose you do a measurement and get a_1 for the value of $\hat{A}$. Then all successive measurements give a_1.

But if you do measurements on a set of identically prepared systems, you will get the distribution.

In classical systems – measurement does not change system.

In QM systems, measurements change the system (if it is not already in an eigenstate of the observable being measured).

Consider three wavefunctions that differ only in signs of contributions of eigenstates

$$\psi_1 = \frac{\sqrt{11}}{4}\phi_1 + \frac{1}{4}\phi_2 + \frac{1}{2}\phi_3(x)$$

$$\psi_4 = \frac{\sqrt{11}}{4}\phi_1 - \frac{1}{4}\phi_2 + \frac{1}{2}\phi_3(x)$$

$$\psi_5 = \frac{\sqrt{11}}{4}\phi_1 - \frac{1}{4}\phi_2 - \frac{1}{2}\phi_3(x)$$

multiple measurements on each give same probabilities of $\phi_1,\ \phi_2,\ \phi_3$

Cannot fully determine a quantum mechanical wavefunction by measurements

This is a consequence of the fact it is $\psi^*\psi$ that we can measure rather than ψ .

The order of operators can matter

$$\hat{x}\hat{p}_x \sin x = x\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)\sin x$$

$$= \frac{\hbar}{i} x \cos x$$

$$\hat{p}_x \hat{x} \sin x = \frac{\hbar}{i}\left[\sin x + x \cos x\right]$$

This is the origin of the uncertainty principle between $\hat{x}$ and $\hat{p}_x$.

Cannot know precisely both $\hat{x}$ and $\hat{p}_x$

So the order matters

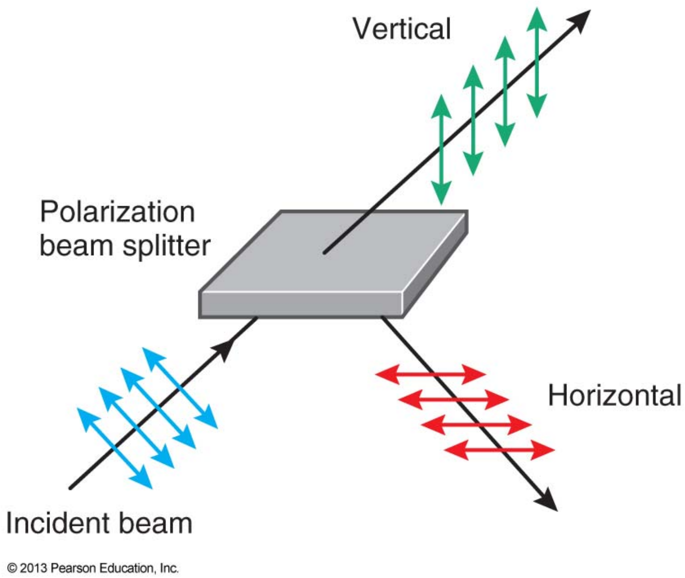
In classical world can know x and p_x precisely, and order of measurements does not matter

What about $\hat{p}_y \hat{x}$ and $\hat{x}\hat{p}_y$?

We are all familiar with polarizing filters

Simple filters transmit vertically polarized light and absorb horizontally polarized light

One can also build filters that transmit vertically polarized light and reflect horizontally polarized light (such as shown below)



Random polarization

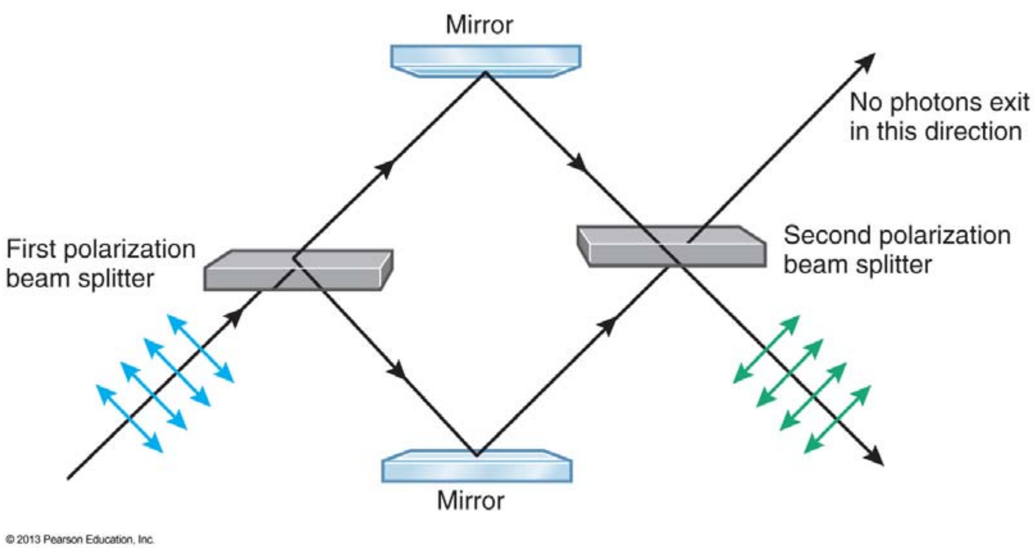
Now consider doing the experiment in the case of a single photon

One sees either || or | polarization (transmitted *vs.* reflected) after passing through beam splitter

Photon is forced an eigenfunction of the polarization operator by the measurement (text explanation)

I believe it is more proper to consider the measurement as causing detection in either the transmission or reflection channels (the polarization then follows)

Now consider a variation of the experiment with two beam splitters.



Photon comes out with original polarization.

Only do measurement after passing through 2nd beam splitter.

Photon emerges with original polarization.

Text coveys the idea that the photon leaving first splitter is in superposition state along both paths.

I believe it leaves in a superposition of transmitted (perpendicular) and reflected (parallel) because of the way the beam splitter works

Link to a java applet for two coupled pendula
http://www.walter-fendt.de/ph14e/cpendula.htm

Note that this system has two natural frequencies and can be prepared in either of these or in a superposition of the two natural motions.