CHEM 1410 Homework 1 Solutions

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Problem 1: Q.1.1

According to the Bohr model of the hydrogen atom, the frequency of a photon emitted due to the transition $n_2 \to n_1$ is

$$\nu_{n_2 \to n_1} = \frac{m_e e^4}{8\epsilon_0^2 h^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

where $n_2 > n_1$. Let's examine a hypothetical case where $n_1 = 1$, the ground state, and n_2 approaches ∞. Then

$$\lim_{n_2 \to \infty} \frac{m_e e^4}{8\epsilon_0^2 h^3}\left(\frac{1}{1^2} - \frac{1}{n_2^2}\right) = \frac{m_e e^4}{8\epsilon_0^2 h^3}.$$

It is clear that as n_2 approaches ∞, the second term inside the parentheses goes to 0, and the photon frequency approaches a constant value, defining an upper limit in the atomic spectrum.

Problem 2: Q.1.10

The classical description of the spectral density distribution of a blackbody results in a divergence as the frequencies approach the ultraviolet region of the electromagnetic spectrum. This deviation is appropriately called the "ultraviolet catastrophe."

Problem 3

Two answers are acceptable for this problem and will receive full credit:
(1) The Bohr model for the hydrogen atom depends only on the electron's mass as written in the textbook. Thus, the nuclear mass has no effect on the electronic spectrum of H vs. D, leading to identical spectra.
(2) The Bohr model for the hydrogen atom actually depends on the reduced mass of the two-body system, which includes the nuclear mass. This would lead to D emitting higher frequency photons than H.

Problem 4: P.1.11

Solving for velocity v in the de Broglie relation yields

$$p = \frac{h}{\lambda} \;\Rightarrow\; mv = \frac{h}{\lambda} \;\Rightarrow\; v = \frac{h}{m\lambda}.$$

Solving for temperature T in the equation for v_{rms} gives

$$v_{rms} = \sqrt{\frac{3k_B T}{m}} \;\Rightarrow\; T = \frac{m v_{rms}^2}{3k_B}.$$

Substituting in the expression for velocity v into the expression for temperature T gives

$$T = \frac{h^2}{3k_B\lambda^2 m} \tag{1}$$

Using Eq. 1, one can calculate the gas temperatures of He and Ar by using their atomic mass as follows.

$$T_{\text{He}} = \frac{(6.626 \times 10^{-34} kg\ m^2\ s^{-1})^2}{3(1.381 \times 10^{-23} kg\ m^2\ s^{-2}\ K^-1)(2.5 \times 10^{-10} m)^2(6.646 \times 10^{-27} kg)} = \mathbf{25.512}\boldsymbol{K}$$

$$T_{\text{Ar}} = \frac{(6.626 \times 10^{-34} kg\ m^2\ s^{-1})^2}{3(1.381 \times 10^{-23} kg\ m^2\ s^{-2}\ K^-1)(2.5 \times 10^{-10} m)^2(6.634 \times 10^{-26} kg)} = \mathbf{2.556}\boldsymbol{K}$$

The gas temperature decreases by one order of magnitude going from He to Ar. Thus, one expects the gas temperature of Xe to be much lower than Ar, and thus prohibitively low for a beam experiment.