HW # 3 Answers Chem 1410

4-5. Which of the following are acceptable state functions?

a) $\frac{1}{x}$ $(0, \infty)$ No, $\frac{1}{x} \to \infty$ as $x \to 0$

b) $e^{-2x} \sinh x$ $(0, \infty)$

$$e^{-2x} \sinh x = e^{-2x}\left(\frac{e^{x} - e^{-x}}{2}\right) = \frac{e^{-x} - e^{-3x}}{2}$$

Yes

c) $e^{-x} \cos x$ $(0, \infty)$ Yes

d) e^{x} $(-\infty, \infty)$ No $e^{x} \to \infty$ as $x \to \infty$

4-14. Do the following pairs of operators commute?

a) $\frac{d}{dx}$, $\frac{d^2}{dx^2} + 2\frac{d}{dx}$, yes

$$\frac{d}{dx}\left(\frac{d^2}{dx^2} + 2\frac{d}{dx}\right) = \frac{d^3}{dx^3} + 2\frac{d^2}{dx^2} = \left(\frac{d^2}{dx^2} + 2\frac{d}{dx}\right)\frac{d}{dx}$$

b) x, $\frac{d}{dx}$, NO

$$x\frac{df}{dx} \neq \frac{d}{dx} xf = f + x\frac{df}{dx}$$

c) SQR, SQRT, NO

Let $x = -2$

$$\text{SQRT}(\text{SQR } x) = \sqrt{4} = \pm 2$$

$$\text{SQR}(\text{SQRT } x) = \left(\sqrt{-2}\right)^2 = \left(\pm\sqrt{2}\, i\right)^2$$

$$= -2$$

d) $x^2 \dfrac{d}{dx}$, $\dfrac{d^2}{dx^2}$ NO

$$x^2 \frac{d}{dx}\frac{d^2 f}{dx^2} = x^2 \frac{d^3 f}{dx^3}$$

$$\frac{d^2}{dx^2}\left(x^2 \frac{d}{dx}\right) f = \frac{d}{dx}\frac{d}{dx}\left(x^2 \frac{df}{dx}\right) = \frac{d}{dx}\left(2x\frac{df}{dx} + x^2 \frac{d^2 f}{dx^2}\right)$$

$$= 2\frac{df}{dx} + 2x\frac{d^2 f}{dx^2} + 2x\frac{d^2 f}{dx^2} + x^2 \frac{d^3 f}{dx^3}$$

$$= 2\frac{df}{dx} + 4x\frac{d^2 f}{dx^2} + x^2\frac{d^3 f}{dx^3}$$

4-17 Show $\hat{L}_x \hat{L}_y - \hat{L}_y \hat{L}_x = i\hbar \hat{L}_z$

$$\hat{L}_x = \frac{\hbar}{i}\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right), \quad \hat{L}_y = \frac{\hbar}{i}\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right)$$

$$\hat{L}_x \hat{L}_y = -\hbar^2\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right)$$

$$= -\hbar^2\left(y\frac{\partial}{\partial x} + \underline{yz\frac{\partial}{\partial z}\frac{\partial}{\partial x}} - z^2\underline{\frac{\partial}{\partial y}\frac{\partial}{\partial x}} + \underline{zx\frac{\partial}{\partial y}\frac{\partial}{\partial z}} - \underline{yx\frac{\partial^2}{\partial z^2}}\right)$$

$$\hat{L}_y \hat{L}_x = -\hbar^2\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right)\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)$$

$$= -\hbar^2\left(\underline{zy\frac{\partial}{\partial x}\frac{\partial}{\partial z}} - z^2\underline{\frac{\partial}{\partial x}}\frac{\partial}{\partial y} - \underline{xy\frac{\partial^2}{\partial z^2}} + x\frac{\partial}{\partial y} + \underline{xz\frac{\partial}{\partial z}\frac{\partial}{\partial y}}\right)$$

$$\hat{L}_x \hat{L}_y - \hat{L}_y \hat{L}_x = -\hbar^2\left(y\frac{\partial}{\partial x} - x\frac{\partial}{\partial y}\right)$$

$$= \frac{\hbar}{i}\frac{\hbar}{i}\left(-x\frac{\partial}{\partial y} + y\frac{\partial}{\partial x}\right) = -\frac{\hbar}{i}\hat{L}_z = \hbar i \hat{L}_z$$

underlined quantities cancel out.

The proof for the other two commutators is similar, and, in fact, follows from the $x \leftrightarrow y \leftrightarrow z$ permutation of the variables.