

1

$$E = nh\nu = nh\frac{c}{\lambda} \Rightarrow 1 = n \times 6.626 \times 10^{-34} \times \frac{2.998 \times 10^8}{550 \times 10^{-9}} \Rightarrow 2.77 \times 10^{18} \text{ photons}$$

2

$$E_e = h\nu - \phi$$

$$h\nu = 6.626 \times 10^{-34} \times \frac{2.998 \times 10^8}{700 \times 10^{-9}} = 2.84 \times 10^{-19} \text{ J}$$

$$\phi = 2.14 \text{ eV} \times 1.602 \times 10^{-19} \frac{\text{J}}{\text{eV}} = 3.43 \times 10^{-19} \text{ J}$$

Since $\phi > h\nu$, no electron will be emitted.

3

$$d = 0.375 \text{ cm}, \quad b = ?$$

$$\sin\theta = \frac{d}{\sqrt{d^2 + b^2}} = \frac{\lambda}{a}$$

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}}{9.109 \times 10^{-31} \times 4.75 \times 10^4} = 1.53 \times 10^{-8} \text{ m}$$

$$\frac{3.75 \times 10^{-3}}{\sqrt{(3.75 \times 10^{-3})^2 + b^2}} = \frac{1.53 \times 10^{-8}}{2.35 \times 10^{-7}} \Rightarrow 5.76 \times 10^{-2} = \sqrt{(3.75 \times 10^{-3})^2 + b^2}$$

$$b = 5.75 \text{ cm}$$

4

In a set of wave functions, only those that satisfy the eigen value equation are considered eigen functions.

5

$$\frac{\partial^2}{\partial x^2} e^{-i(3x+2y)} = 9i^2 e^{-i(3x+2y)} \Rightarrow EV = -9$$

$$\frac{1}{x}(x^2 + y^2) \frac{\partial}{\partial x} (x^2 + y^2)^{\frac{1}{2}} = \frac{1}{x}(x^2 + y^2) \times \frac{1}{2} 2x(x^2 + y^2)^{-\frac{1}{2}} = (x^2 + y^2)^{\frac{1}{2}} \Rightarrow EV = 1$$

$$\begin{aligned}
& \left(\sin\theta \frac{d}{d\theta} \left(\sin\theta \frac{d}{d\theta} \right) + 6 \sin^2 \theta \right) (\sin\theta \cos\theta) = \left(\sin\theta \frac{d}{d\theta} \left(\sin\theta \frac{d}{d\theta} \right) \right) (\sin\theta \cos\theta) + 6 \sin^3 \theta \cos\theta \\
& \Rightarrow \sin\theta \frac{d}{d\theta} (\sin\theta) (\cos^2 \theta - \sin^2 \theta) + 6 \sin^3 \theta \cos\theta \\
& = \sin\theta \frac{d}{d\theta} (\sin\theta) (1 - 2\sin^2 \theta) + 6 \sin^3 \theta \cos\theta \\
& = \sin\theta \frac{d}{d\theta} (\sin\theta - 2\sin^3 \theta) + 6 \sin^3 \theta \cos\theta \\
& = \sin\theta (\cos\theta - 6 \cos\theta \sin^2 \theta) + 6 \sin^3 \theta \cos\theta = \sin\theta \cos\theta \Rightarrow EV = 1
\end{aligned}$$

Note that: $\cos^2 \theta - \sin^2 \theta = (1 - \sin^2 \theta) - \sin^2 \theta = 1 - 2\sin^2 \theta$.

6

$$\int_{-1}^1 a_0^2 dx = a_0^2 x \Big|_{-1}^1 \Rightarrow a_0^2 - (-a_0^2) = 2a_0^2$$

$$2a_0^2 = 1 \Rightarrow a_0 = \pm \frac{1}{\sqrt{2}}$$

$$\begin{aligned}
\int_{-1}^1 a_0(a_1 + b_1 x) dx & \Rightarrow \int_{-1}^1 (a_0 a_1 + a_0 b_1 x) dx \Rightarrow a_0 a_1 x + a_0 b_1 \frac{x^2}{2} \Big|_{-1}^1 \\
& \Rightarrow \left(a_0 a_1 + \frac{1}{2} a_0 b_1 \right) - \left(-a_0 a_1 + \frac{1}{2} a_0 b_1 \right) = 2a_0 a_1
\end{aligned}$$

$$\frac{2}{\sqrt{2}} a_1 = 0 \Rightarrow a_1 = 0$$

$$\int_{-1}^1 (b_1 x)(b_1 x) dx \Rightarrow \int_{-1}^1 b_1^2 x^2 dx \Rightarrow b_1^2 \frac{x^3}{3} \Big|_{-1}^1 \Rightarrow \left(\frac{1}{3} b_1^2 \right) - \left(-\frac{1}{3} b_1^2 \right) = \frac{2}{3} b_1^2$$

$$\frac{2}{3} b_1^2 = 1 \Rightarrow b_1 = \pm \sqrt{\frac{3}{2}}$$

$$\begin{aligned}
\int_{-1}^1 a_0(a_2 + b_2 x + c_2 x^2) dx & \Rightarrow \int_{-1}^1 (a_0 a_2 + a_0 b_2 x + a_0 c_2 x^2) dx \Rightarrow a_0 a_2 x + a_0 b_2 \frac{x^2}{2} + a_0 c_2 \frac{x^3}{3} \Big|_{-1}^1 \\
& \Rightarrow \left(a_0 a_2 + \frac{1}{2} a_0 b_2 + \frac{1}{3} a_0 c_2 \right) - \left(-a_0 a_2 + \frac{1}{2} a_0 b_2 - \frac{1}{3} a_0 c_2 \right) = 2a_0 a_2 + \frac{2}{3} a_0 c_2
\end{aligned}$$

$$2a_0 a_2 + \frac{2}{3} a_0 c_2 = 0 \Rightarrow \frac{2}{\sqrt{2}} a_2 + \frac{2}{3\sqrt{2}} c_2 = 0 \Rightarrow a_2 = -\frac{1}{3} c_2$$

$$\begin{aligned}
\int_{-1}^1 (a_1 + b_1x)(a_2 + b_2x + c_2x^2)dx &\Rightarrow \int_{-1}^1 (a_1a_2 + a_1b_2x + a_1c_2x^2 + a_2b_1x + b_1b_2x^2 + b_1c_2x^3) dx \\
&\Rightarrow a_1a_2x + a_1b_2\frac{x^2}{2} + a_1c_2\frac{x^3}{3} + a_2b_1\frac{x^2}{2} + b_1b_2\frac{x^3}{3} + b_1c_2\frac{x^4}{4}\Big|_{-1}^1 \\
&\Rightarrow \left(a_1a_2 + \frac{1}{2}a_1b_2 + \frac{1}{3}a_1c_2 + \frac{1}{2}a_2b_1 + \frac{1}{3}b_1b_2 + \frac{1}{4}b_1c_2\right) \\
&\quad - \left(a_1a_2 + \frac{1}{2}a_1b_2 - \frac{1}{3}a_1c_2 + \frac{1}{2}a_2b_1 - \frac{1}{3}b_1b_2 + \frac{1}{4}b_1c_2\right) = 2a_1a_2 + \frac{2}{3}a_1c_2 + \frac{2}{3}b_1b_2 \\
&\quad 2a_1a_2 + \frac{2}{3}a_1c_2 + \frac{2}{3}b_1b_2 = 0 \Rightarrow \frac{2}{3}b_1b_2 = 0 \Rightarrow b_2 = 0
\end{aligned}$$

$$\begin{aligned}
\int_{-1}^1 (a_2 + c_2x^2)(a_2 + c_2x^2)dx &\Rightarrow \int_{-1}^1 (a_2^2 + 2a_2c_2x^2 + c_2^2x^4)dx \Rightarrow a_2^2x + a_2c_2\frac{2x^3}{3} + c_2^2\frac{x^5}{5}\Big|_{-1}^1 \\
&\Rightarrow \left(a_2^2 + \frac{2}{3}a_2c_2 + \frac{1}{5}c_2^2\right) - \left(-a_2^2 - \frac{2}{3}a_2c_2 - \frac{1}{5}c_2^2\right) = \left(2a_2^2 + \frac{4}{3}a_2c_2 + \frac{2}{5}c_2^2\right)
\end{aligned}$$

$$2a_2^2 + \frac{4}{3}a_2c_2 + \frac{2}{5}c_2^2 = 1 \Rightarrow \frac{2}{9}c_2^2 - \frac{4}{9}c_2^2 + \frac{2}{5}c_2^2 = 1 \Rightarrow \left(-\frac{2}{9} + \frac{2}{5}\right)c_2^2 = 1 \Rightarrow c_2 = \pm \sqrt{\frac{45}{8}} = \pm \frac{3}{2}\sqrt{\frac{5}{2}}$$

$$a_2 = -\sqrt{\frac{45}{72}} = \pm \frac{1}{2}\sqrt{\frac{5}{2}}$$

$$\pm \frac{1}{\sqrt{2}}, \pm \sqrt{\frac{3}{2}}x, -\sqrt{\frac{45}{72}} \pm \sqrt{\frac{45}{8}}x^2$$