



$$Q_1 = x_c - x_b = R_{bc}$$

$$Q_2 = \frac{1}{2}(x_b + x_c) - x_a$$

$$Q_3 = \frac{1}{3}(x_a + x_b + x_c)$$

$$x_c = \frac{1}{2}Q_1 + \frac{1}{3}Q_2 + Q_3$$

$$x_b = -\frac{1}{2}Q_1 + \frac{1}{3}Q_2 + Q_3$$

$$x_a = -\frac{2}{3}Q_2 + Q_3$$

$$H = \frac{Pa^2}{2m} + \frac{Pb^2}{2m} + \frac{Pc^2}{2m} + V(x_a, x_b, x_c) \rightarrow \frac{1}{2m} \left(\frac{1}{2} P_1^2 + \frac{2}{3} P_2^2 + 3P_3^2 \right) + V(R_{ab}, R_{bc}, R_{ac})$$

$$\left. \begin{aligned} \frac{\partial P_1}{\partial t} &= -\frac{\partial H}{\partial Q_1} \\ \frac{\partial P_2}{\partial t} &= -\frac{\partial H}{\partial Q_2} \\ \frac{\partial P_3}{\partial t} &= -\frac{\partial H}{\partial Q_3} \end{aligned} \right\} \frac{\partial P_i}{\partial t} = -\frac{\partial H}{\partial Q_i}$$

$$\left. \begin{aligned} \frac{\partial Q_1}{\partial t} &= \frac{1}{2m} P_1 \\ \frac{\partial Q_2}{\partial t} &= \frac{1}{2m} \left(\frac{4}{3} P_2 \right) \\ \frac{\partial Q_3}{\partial t} &= \frac{1}{2m} (6P_3) \end{aligned} \right\} \frac{\partial Q_i}{\partial t} = \frac{\partial H}{\partial P_i}$$

Hamiltonian approach to defining the quantities needed to solve the differential equation.

$$V(R_{ab}, R_{bc}, R_{ac}) = V(x_1, x_2, x_3) = v(Q_1, Q_2, Q_3)$$

Suppose we have a system of Lennard-Jones atoms.

$$V = 4\epsilon \left(\frac{1}{R_{ab}^{12}} - \frac{1}{R_{ab}^6} \right) + 4\epsilon \left(\frac{1}{R_{bc}^{12}} - \frac{1}{R_{bc}^6} \right) + 4\epsilon \left(\frac{1}{R_{ac}^{12}} - \frac{1}{R_{ac}^6} \right)$$

$$\frac{\partial V}{\partial Q_1} = \frac{\partial V_{ab}}{\partial R_{ab}} \frac{\partial R_{ab}}{\partial Q_1} + \frac{\partial V_{bc}}{\partial R_{bc}} \frac{\partial R_{bc}}{\partial Q_1} + \frac{\partial V_{ac}}{\partial R_{ac}} \frac{\partial R_{ac}}{\partial Q_1}, \text{ etc.}$$