$$\left\langle \vec{J} \right\rangle = \vec{\sigma} \cdot \vec{E} + \quad \Big| \quad \Rightarrow$$

steady (DC) conductance

conductivity ($\vec{\sigma}$), field ($\vec{E}$)

more general, allowing for time dependence

$$\left\langle J(t) \right\rangle = \int_0^t dt' \underbrace{\vec{\Phi}(t-t')}_{\text{after effect (lag)}} \cdot \vec{E}(t)$$

Fourier-Laplace transform

$$\left\langle \vec{J}_\omega \right\rangle = \vec{\sigma}(\omega) \cdot \vec{E}_\omega$$

$$\left\langle \vec{J}_\omega \right\rangle = \int_0^\infty e^{-i\omega t} \left\langle \vec{J}(t) \right\rangle dt$$

$$\vec{E}_\omega = \int_0^\infty e^{-i\omega t} \vec{E}(t) dt$$

$$\vec{\sigma}(\omega) = \int_0^\infty dt e^{-i\omega t} \Phi(t)$$

The above results are general
(i.e., not limited to the example
of electrical conductivity)

B depends on ext. field, F

$$\langle B(t)\rangle = \int_0^\infty \phi(t-t')F(t')dt'$$

$$\langle B_\omega\rangle = \chi(\omega)F_\omega$$

↖ freq.-dep. susceptibility

Dielectric relaxation of a gas of non-interacting dipoles

$$\vec{P} = \chi(\omega)\vec{E}$$ total polarization of a gas (dipole/unit vol)

$$\vec{D} = \vec{E} + 4\pi\vec{P}$$

$$\vec{D} = \varepsilon\vec{E}$$

$$\phi(t) \text{ real } \rightarrow$$

$$\chi(\omega),\ \varepsilon(\omega)$$ complex

$$\varepsilon(\omega) = 1 + 4\pi\chi(\omega) =$$ dielectric constant

$$\varepsilon(\omega) = \varepsilon'(\omega) - i\varepsilon''(\omega)$$

$$\varepsilon'' = \frac{nC\alpha(\omega)}{\omega}$$ = dielectric loss

n = index of refraction
α = absorption coeff.

$$I = I_0 e^{-\alpha x}$$

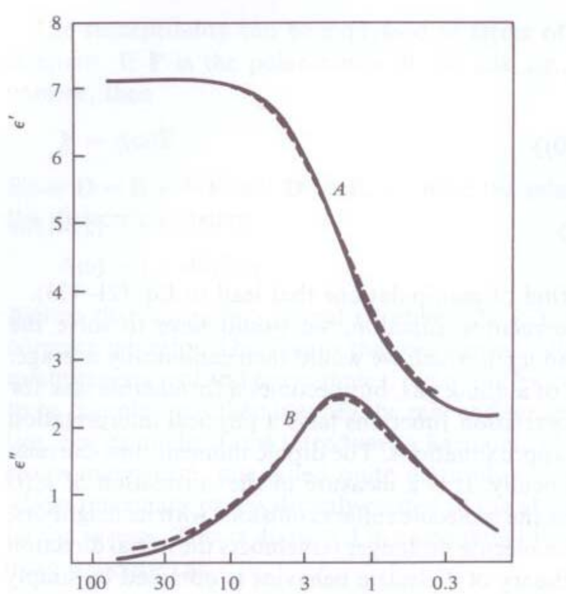
Debye: $\langle u_z(0) u_z(t) \rangle = u_{z0}^2 e^{-|t|/\tau}$

$$\rightarrow \chi(\omega) = \beta\mu_0^2 \left[\frac{1}{1+\omega^2\tau^2} - \frac{i\omega\tau}{1+\omega^2\tau^2} \right]$$

$$\varepsilon' - 1 = 4\pi\beta\mu_0^2 \left(\frac{1}{1+\omega^2\tau^2} \right)$$

$$\varepsilon'' = 4\pi\beta\mu_0^2 \left(\frac{\omega\tau}{1+\omega^2\tau^2} \right)$$

Debye equations



wavelength

(from McQuarrie)

molecules begin to absorb radiation as they lag behind the field

eventually freq so high, molecules cannot respond

Kramers-Kronig relations

$$\chi'(\omega) = \frac{2}{\pi}\int_0^\infty \chi''(\omega')\frac{\omega' d\omega'}{\omega^2 - \omega'^2} d\omega'$$

$$\chi''(\omega) = \int_0^\infty \chi'(\omega')\frac{\omega d\omega'}{\omega^2 - \omega'^2} d\omega'$$

More generally

$$I(\omega)=\frac{\hbar\varepsilon''\omega}{4\pi^2\left(1-e^{-\beta\hbar\omega}\right)}=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left\langle\mu_z\mu_z(t)\right\rangle e^{-i\omega t}dt$$

$$C(t)=\left\langle\mu_z\mu_z(t)\right\rangle$$

Real part *C* $\rightarrow$ even function of *t*
Imag part *C* $\rightarrow$ odd function of *t* vanishes in classical system

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Note: we can express time dependence *via* taylor series

$$\left\langle A(0)A(t)\right\rangle=\left\langle A(0)A(0)\right\rangle+\underbrace{\left\langle A(0)\dot{A}(0)\right\rangle t}_{0}+\left\langle A(0)\ddot{A}(0)\right\rangle\frac{t^2}{2}+\ldots$$

0 ← for equil. system

$$\left\langle A(0)A(t)\right\rangle\approx\left\langle A(0)A(0)\right\rangle-\left\langle\dot{A}(0)\dot{A}(0)\right\rangle\frac{t^2}{2}+\ldots$$

$$\langle u_z u_z(t)\rangle = \int_{-\infty}^{\infty} e^{i\omega t} I(\omega)\, d\omega$$

$$= \sum_{n=0} \frac{(it)^n}{n!} \underbrace{\int \omega^n I(\omega)\, d\omega}_{\text{moments}}$$

moments give information on the lineshape