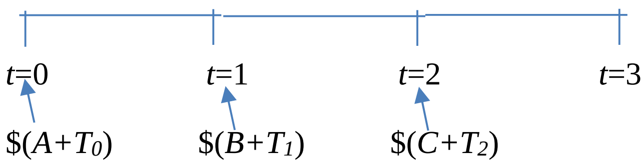
I.E. 2001 OPERATIONS RESEARCH (Spring 2020)
(Solutions to Assignment 2)

<u>Question 5, p. 114</u>

Define A = Dollars invested in investment A at time 0
B = Dollars invested in investment B at time 1
C = Dollars invested in investment C at time 2
T_j = Dollars invested in T-bills at time j, j=0,1,2

NOTE: Time t refers to end of year t



Maximize $1.2C + 1.1T_2$ (Maximize Cash at time 3)

st $T_0 + A = 100$ (Investment at time 0 = cash available now)
$T_1 + B = 0.1A + 1.1T_0$ (Investment at time 1 = cash available end of yr. 1)
$T_2 + C = 1.3A + 1.6B + 1.1T_1$ (Investment at time 2 = cash available end of yr. 2)
$A \le 50$, $B \le 50$, $C \le 50$ (Allowed limits on investments)
$A, B, C, T_1, T_2, T_3 \ge 0$

NOTE: The "=" in the first three constraints can also be replaced with "≤" – WHY?!

<u>Question 13, p. 115</u>

Define W_1= pounds of wheat used in the amount of Feed 1produced and sold
A_1= pounds of alfalfa used in the amount of Feed 1 produced and sold
W_2= pounds of wheat used in the amount of Feed 2 produced and sold
A_2= pounds of alfalfa used in the amount of Feed 2 produced and sold

Maximize Profit = $1.5(W_1+A_1)+1.3(W_2+A_2)- 0.5(W_1+W_2)- 0.4(A_1+A_2)$
$(= W_1+1.1A_1+0.8W_2+0.9A_2)$

st $W_1+W_2 \le 1000$ (Max. Wheat purchase possible)
$A_1 + A_2 \le 800$ (Max. Alfalfa purchase possible)
$W_1/(A_1+W_1) \ge 0.8$, i.e., $0.2W_1- 0.8A_1 \ge 0$ (Wheat % Req. in Feed 1)
$A_2/(A_2+W_2) \ge 0.6$, i.e., $0.4A_2- 0.6W_2 \ge 0$ (Alfalfa % Req. in Feed 2)
$A_1, A_2, W_1, W_2 \ge 0$

<u>Question 34, p. 118</u>

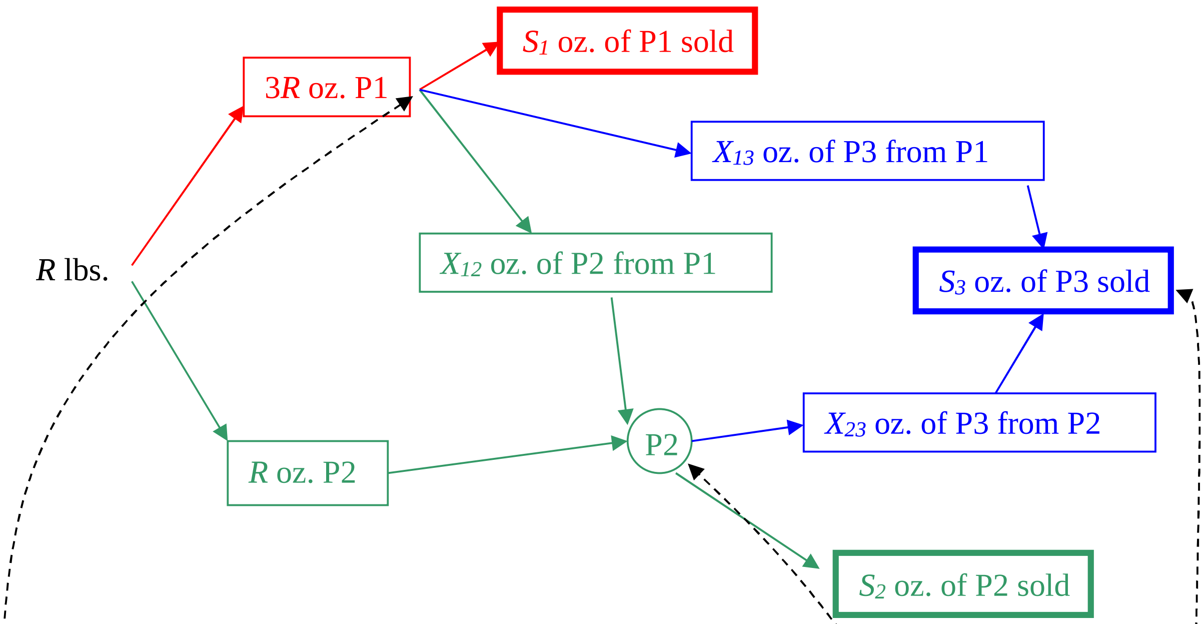
Let L_t = No. of air conditioners made in LA in month t, t=1,2,3
N_t = No. of air conditioners made in NY in month t, t=1,2,3
I_t = No. of air conditioners in inventory at the end of month t, t=1,2,3

Min $400L_1+400L_2+400L_3+350N_1+350N_2+350N_3+100I_1+100I_2+100I_3$
st

$L_1+N_1+200-300=I_1$, i.e., $L_1+N_1-I_1=100$ (period 1 inventory balance)
$L_2+N_2+I_1-400=I_2$, i.e., $L_2+N_2+I_1-I_2= 400$ (period 2 inventory balance)
$L_3+N_3+I_2-500=I_3$, i.e., $L_3+N_3+I_2-I_3= 500$ (period 3 inventory balance)
$1.5L_t \le 420$ for t=1,2,3 (skilled labor availability in LA in each month)
$2N_t \le 420$ for t=1,2,3 (skilled labor availability in NY in each month)
All variables nonnegative.

Question 4, p. 98

Consider the following schematic of the process:



Define R=lbs of raw material used
X_{12}=Ounces of Product 1 processed into Product 2
X_{13}=Ounces of Product 1 processed into Product 3
X_{23}=Ounces of Product 2 processed into Product 3
S_i=Ounces of Product i sold (i=1, 2, 3)

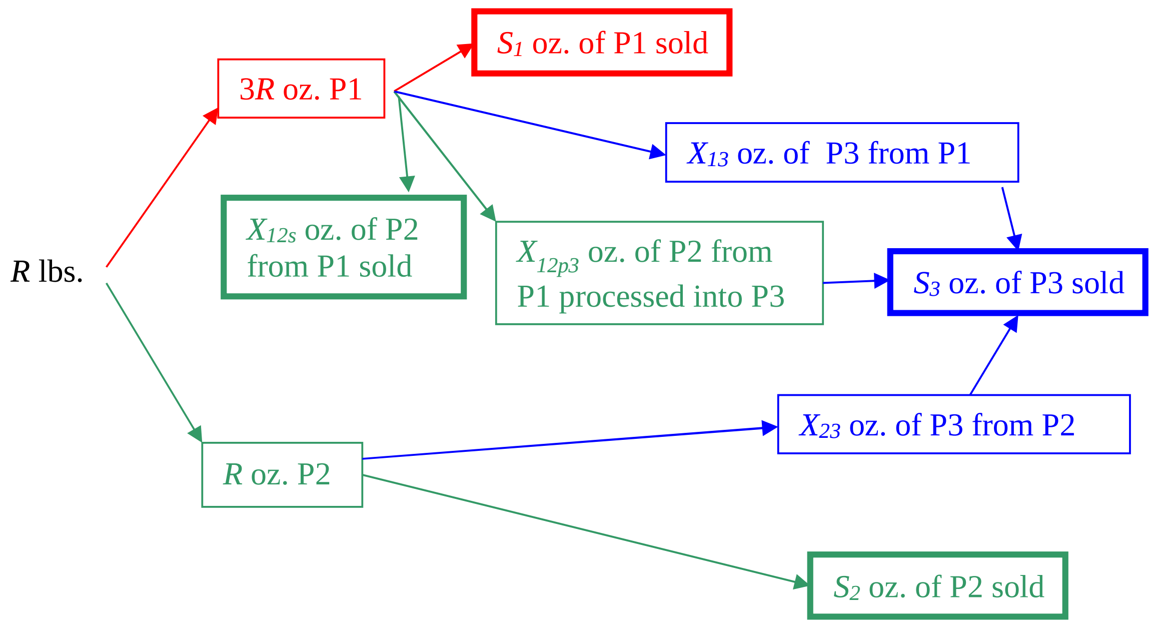
Max $(10S_1+20S_2+30S_3)-(25R)-(1R-1X_{12}-2X_{13}-6X_{23})$
(revenue) – (raw material processing cost) – (product reprocessing costs)

$= 10S_1 + 20S_2 + 30S_3 - 26R - X_{12} - 2X_{13} - 6X_{23}$ (Profits)

st

$S_1 \le 5{,}000$, $S_2 \le 5{,}000$, $S_3 \le 3{,}000$ (maximum sales potential)
$2R + 2X_{12} + 3X_{13} + X_{23} \le 25{,}000$ (labor availability)
$3R = S_1 + X_{12} + X_{13}$ (material balance for Prod 1)
$R + X_{12} = S_2 + X_{23}$ (material balance for Prod 2)
$S_3 = X_{13} + X_{23}$ (material balance for Prod 3)
All variables nonnegative

An alternative (and somewhat more complex…) formulation would be one where you could have separate variables for the amount of P1 that is converted to P2 and sold and for the amount of P1 that is converted to P2 and then converted to P3:



Define R=lbs of raw material used

X_{12s}=Ounces of Product 1 processed into Product 2 and sold

X_{12p3}=Ounces of Product 1 processed into Product 2 and then into Product 3

Then a correct formulation is

Max $10S_1+20S_2+20X_{12s}+30S_3-(25R+1R)-(1X_{12s}+1X_{12p3}+2X_{13})-(6X_{23}+6X_{12p3})$

$= 10S_1+20S_2+19X_{12s}+30S_3-26R-7X_{12p3}-2X_{13}-6X_{23}$ (Profits)

st

$S_1\leq 5{,}000,\ \ S_2+X_{12s}\leq 5{,}000,\ \ S_3\leq 3{,}000$ (maximum sales potential)

$2R+2X_{12s}+2X_{12p3}+3X_{13}+X_{23}\leq 25{,}000$ (labor availability)

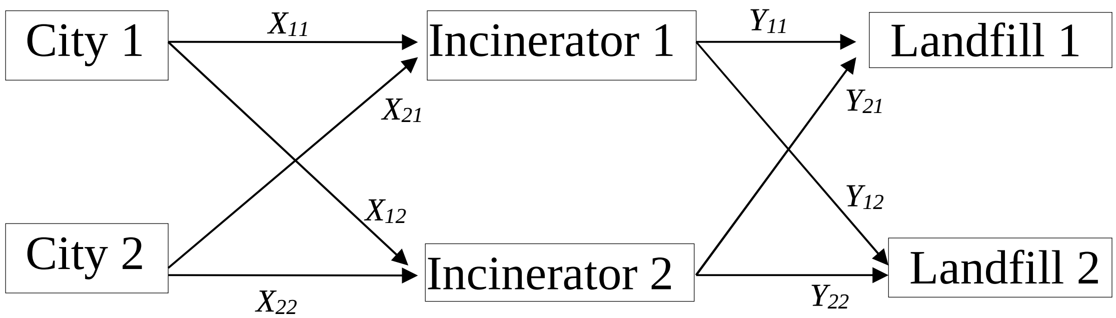
$3R = S_1+X_{12s}+X_{12p3}+X_{13}$ (material balance for Prod 1)

$R = S_2+X_{23}$ (material balance for Prod 2)

$S_3 = X_{13}+X_{12p3}$ (material balance for Prod 3)

All variables nonnegative

Question 50, p. 121-122
The problem may be represented schematically as follows:



Let X_{ij} = Tons of City i waste that is sent to Incinerator j; i=1,2; j=1,2.
Y_{jk} = Tons of debris sent from Incinerator j to Landfill k; j=1,2; k=1,2.

Then the appropriate LP is

Min Z = $40(X_{11}+X_{21}) + 30(X_{12}+X_{22}) +$
$3[30X_{11}+ 5X_{12}+ 36X_{21}+ 42X_{22} + 5Y_{11}+ 8Y_{12}+ 9Y_{21}+ 6Y_{22}]$

s.t.

$X_{11} + X_{12} = 500$ (CITY 1 WASTE MATERIAL BALANCE)
$X_{21} + X_{22} = 400$ (CITY 2 WASTE MATERIAL BALANCE)

$Y_{11} + Y_{12} = 0.2(X_{11}+X_{21})$ (INCINERATOR 1 MATERIAL BALANCE)
$Y_{21} + Y_{22} = 0.2(X_{12}+X_{22})$ (INCINERATOR 2 MATERIAL BALANCE)

$Y_{11} + Y_{21} \leq 200$ (Landfill 1 capacity)
$Y_{12} + Y_{22} \leq 200$ (Landfill 2 capacity)

$X_{11} + X_{21} \leq 500$ (Incinerator 1 limitation)
$X_{12} + X_{22} \leq 500$ (Incinerator 2 limitation)

All $X_{ij}, Y_{ij} \geq 0$