

IE 2001: OPERATION RESEARCH
(Homework Assignment 3: Due February 6, 2020)

1. Formulate Problem 51, page 122. (Silvco)
2. Formulate Problem 52, page 122. (Paper Recycling)

Again, for both problems, I suggest that you **draw schematic diagrams or pictures** of the system being modeled - this will make things easier...

3. A company supplies fertilizers to its customers on a make-to-order basis by blending appropriate amounts of “stock” fertilizers. These stock fertilizers are purchased direct from a factory in 100 lb. bags and either a whole bag or a part of it can be used in the final blend. Customer orders are specified as a triple of percentages N - P - K where N , P and K are (respectively) the *minimum* desired percentages of nitrogen, phosphorus and potassium in the fertilizer. An order has just been received for 1000 lbs of 17 - 14 - 10 fertilizer: the customer wants at least 17% of the fertilizer’s weight to be nitrogen, at least 14% to be phosphorus and at least 10% to be potassium. There are three “stock” fertilizer types and the company will use a total of ten 100-lb bags of these to fill this order: Type 1 is a 50 - 20 - 5 fertilizer, Type 2 is a 0 - 15 - 20 fertilizer and Type 3 is a 10 - 10 - 10 fertilizer (for stock fertilizers N - P - K values represent *exact* percentage compositions in the 100 lb. bag). The costs of a 100 lb. bag of Type 1, Type 2 and Type 3 are \$90, \$20 and \$30 respectively.
 - a) Formulate a linear program to come up with the least expensive way to satisfy the order.
 - b) Convert the formulation to a 2 variable problem so that it can be solved graphically (HINT: Eliminate one of the variables – say X_3 ...). Sketch the feasible region and compute the coordinates of its extreme points.
 - c) Graphically find the optimal blend and the corresponding minimum cost. Sketch an isocost line on your graph. Which constraints are active and which ones inactive at the optimum? By how much do the contents of the three nutrients in the mix exceed the minimal requirement of the customer?
 - d) Suppose the price per bag of Types 1, 2 and 3 of “stock” fertilizer go down to \$85, \$10 and \$25 respectively. Repeat (c); if there is more than one optimum solution, characterize the complete set of optimum solutions. What is the optimum value of the objective?

4. Consider the following LP:

$$\begin{aligned} \text{Min } & -2x_1 + 6x_2 \\ \text{st } & x_1 + x_2 \geq 2 \\ & -x_1 + x_2 \leq 1 \\ & x_1 \geq 0, \quad x_2 \geq 0 \end{aligned}$$

- a) Sketch the feasible region. What can you say about it?
 - b) Does the problem have an optimal solution? If so, where? If it does not, then why not?
 - c) How (if at all) do your answers change if the objective is to *maximize* the same function?
5. Read Chapter 2 in your text. Focus on Sections 2-1, 2-2, 2-3 (especially this one), and 2-5. This is stuff that you should (hopefully...) have already seen in an undergraduate linear algebra class and should be in the nature of a review. If you have time, work out a couple of problems at the end of each section - I'd like you to all become re-acquainted with vectors, matrices and systems of linear equations. You should begin reviewing these topics now so that you don't get lost when we hit Chapters 4 and (especially) 6.