

I.E.2001 OPERATIONS RESEARCH

(Homework Assignment No. 4: Due Feb. 13, 2020)

I. Restate the following LP so that it is in the standard form - do NOT try solving it...

$$\begin{aligned}
 & \text{Max} \quad -3X_1 + X_2 - 2X_3 + X_4 \\
 \text{st} \quad & -4X_1 + X_2 + X_3 \geq 4 \\
 & 3X_1 - X_2 + 2X_3 \leq -6 \\
 & X_2 + 4X_3 = X_4 - 1 \\
 & 2X_1 - X_2 = -X_3 \\
 & X_1 \text{ unrestricted; } X_2, X_3, X_4 \geq 0
 \end{aligned}$$

II. Consider a linear program where $6X_1 + 4X_2$ is maximized subject to the four constraints

$$\begin{aligned}
 1) \quad & 8X_1 + 15X_2 \geq 30 \\
 2) \quad & -X_1 + X_2 \leq 5 \\
 3) \quad & 2X_1 + 3.75X_2 \leq 30 \\
 4) \quad & 1.5X_1 + X_2 \leq 15 \quad X_1, X_2 \geq 0
 \end{aligned}$$

- a) Graph (neatly...) the feasible region.
- b) Calculate the total number of basic solutions that could exist. How many of these actually exist? Explain any discrepancy.
- c) How many basic feasible solutions does the problem have?
- d) Mark the basic (both infeasible and feasible) solutions on the graph, and calculate the values of all variables (including slacks) at each of these basic solutions.
- e) The intermediate tableau shown below was obtained when solving this LP by the Simplex method. Continue the process and show that the optimum value and solution are found at the next iteration. Do you see anything that is special, unusual, or different about the final (optimal) tableau compared to others you have seen in class so far?

Basic	Z	X_1	X_2	S_1	S_2	S_3	S_4	RHS
Z	1	0	7.25	-0.75	0	0	0	22.5
X_1	0	1	1.875	-0.125	0	0	0	3.75
S_2	0	0	2.875	-0.125	1	0	0	8.75
S_3	0	0	0	0.25	0	1	0	22.5
S_4	0	0	-1.8125	0.1875	0	0	1	9.375

III. Use the simplex method to solve the following LP

$$\begin{aligned}
 & \text{Minimize} \quad Z = -3X_1 + X_2 - 3X_3 - 4X_4 \\
 \text{st} \quad & X_1 + 7X_2 + 3X_3 + 7X_4 \leq 46 \\
 & 3X_1 - X_2 + X_3 + 2X_4 \leq 8 \\
 & 2X_1 + 3X_2 - X_3 + X_4 \leq 10 \\
 & X_1, X_2, X_3, X_4 \geq 0
 \end{aligned}$$