# I.E. 2001 OPERATIONS RESEARCH

(*Homework Assignment 5: Due Thursday Feb. 20, 2020*)

**Question 1.** Consider the tableau below that corresponds to some intermediate iteration of the simplex method applied to an LP problem. Then answer the questions that follow.

| $Z$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | RHS |
|---|---|---|---|---|---|---|---|
| 1 | 170 | 0 | 0 | 25 | -20 | 0 | 15,550 |
| 0 | 3 | 0 | 1 | ½ | -1 | 0 | 65 |
| 0 | 1 | 1 | 0 | 0 | ½ | 0 | 205 |
| 0 | -5 | 0 | 0 | -1 | 2 | 1 | 480 |

a) Which variables are nonbasic, which ones are basic, and what are their respective values?
b) If I were minimizing the objective, which nonbasic variables are legitimate candidates to enter the basis? How about if I were maximizing?
c) Suppose I decided to enter $x_4$ into the basis and increase its value by 100 units. Without doing any pivoting, can you say what the new objective value will be? **Explain clearly**.
d) Corresponding to this 100 unit increase in $x_4$ what are the adjustments that need to be made to the values of the current basic variables in order to maintain feasibility? **Explain clearly**.
e) What is the **maximum** amount of increase possible in the value of $x_4$? **Explain clearly**.
f) Again, without doing any pivoting, can you say what the value of the objective will be after the next iteration is completed?

**Question 2**. Answer Question 4 on page 213 of the text.

**Question 3.** Solve by the Two-Phase method:

$$\begin{aligned} \text{Minimize } Z = 30X_1 + 20X_2 \\ \text{st} \quad X_1 &\geq 4 \\ 2X_1 + X_2 &= 20 \\ X_1 + 2X_2 &\geq 19 \\ X_1, X_2 &\geq 0. \end{aligned}$$

**Question 4**. Answer Question 6 on page 213 of the text. You can use either the Big-M or the two-phase method – you don't need to do it both ways.

**Question 5.** Answer Question 18 on page 214-215 of the text.

**Question 6.** Consider the following LP problem:

$$\begin{aligned} \text{Maximize } Z = 6X_1 + 9X_2 \\ \text{st} \quad X_1 + 4X_2 &\leq 8 \\ X_1 + 2X_2 &\leq 4 \\ X_1, X_2 &\geq 0. \end{aligned}$$

Suppose we use the simplex method, but with the following rule to break ties when we have more than one variable that could be removed from the basis (i.e., for the <u>leaving</u> variable): "*Always pick the leaving variable as the one that is higher up in the tableau.*" Show that the problem is <u>temporarily</u> degenerate. Sketch the feasible region and clearly explain what happens.