I.E. 2001 OPERATIONS RESEARCH

(Solutions to Assignment 6)

QUESTION 1

Output is shown below; the variable definitions are as stated in the solutions to Assignment 3 that were posted earlier...

```
  MIN      50 X1 + 70 X2 + 25 RD + 25 R1 + 25 R2 + 25 R3
  SUBJECT TO
    2)   0.3 X1 + 0.2 X2 - RD - D =    0
    3)   0.3 X1 + 0.2 X2 - R1 - Y1 =    0
    4)   0.2 X1 + 0.25 X2 - R2 - Y2 =    0
    5)   0.15 X1 + 0.2 X2 - R3 - Y3 =    0
    6)   0.25 RD + 0.3 R1 + Y1 >=   3000
    7)   0.15 RD + 0.3 R1 + 0.4 R2 + Y2 >=   3000
    8)   0.2 RD + 0.2 R1 + 0.3 R2 + 0.5 R3 + Y3 >=   2000
    9)   0.05 X1 + 0.15 X2 + 0.1 RD + 0.2 R1 + 0.3 R2 + 0.5 R3 >=
1000
   10)   X1 + X2 + RD + R1 + R2 + R3 <=   20000
  END

 LP OPTIMUM FOUND AT STEP      8
        OBJECTIVE FUNCTION VALUE

        1)      641725.3

  VARIABLE        VALUE          REDUCED COST
        X1     10563.380000           .000000
        X2          .000000         16.280810
        RD      3169.014000           .000000
        R1      1373.240000           .000000
        R2          .000000        129.841500
        R3          .000000         25.000000
         D          .000000         11.003520
        Y1      1795.775000           .000000
        Y2      2112.676000           .000000
        Y3      1584.507000           .000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)          .000000        -11.003520
        3)          .000000        -39.172530
        4)          .000000       -174.735900
        5)          .000000           .000000
        6)          .000000        -39.172530
        7)          .000000       -174.735900
        8)        492.957800          .000000
        9)        119.718300          .000000
       10)       4894.367000          .000000
 NO. ITERATIONS=       8
```

QUESTION 2

The LINDO output is shown below:

LP OPTIMUM FOUND AT STEP 3

OBJECTIVE FUNCTION VALUE

1) 90.00000

VARIABLE	VALUE	REDUCED COST
X1	.000000	27.500000
X2	3.000000	.000000
X3	1.000000	.000000
X4	.000000	50.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	50.000000	.000000
3)	.000000	-2.500000
4)	.000000	-7.500000
5)	5.000000	.000000

NO. ITERATIONS= 3

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X1	50.000000	INFINITY	27.500000
X2	20.000000	18.333330	5.000000
X3	30.000000	10.000000	30.000000
X4	80.000000	INFINITY	50.000000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	800.000000	INFINITY	50.000000
3	6.000000	0.800000	2.857143
4	10.000000	1.333333	4.000000
5	8.000000	5.000000	INFINITY

1a) From the *LINDO* output, the shadow prices are π_1=0, π_2=-2.5, and π_4=0 for constraints 1, 2 and 4 respectively. These are interpreted as follows: each one unit increase in the RHS b_1 from its current value of 800 will increase profits by 0 units, assuming that the increase in b_1 does not change the basis, each one unit increase in the RHS b_2 from its current value of 6 will improve the objective by -2.5 units, i.e., cause it to **increase** by 2.5 units, assuming that the increase in b_2 does not change the basis, and each one unit increase in the RHS b_4 from its current value of 8 will increase profits by 0 units, assuming that the increase in b_4 does not change the basis. Note that this is intuitively sensible. Constraint 1 is slack at the optimum, therefore increasing its RHS isn't going to be improve the objective since the resource is not fully utilized at the optimum. Similarly Constraint 4 is "more than" satisfied since it has a positive excess associated with it and a 1 unit increase isn't going to change the basis so that this excess continues to exist and there is no reason to expect any improvement. Constraint 2 on the other hand **is active.** It is a ≥ constraint and increasing its RHS makes it more difficult to satisfy (since the feasible region shrinks); thus the objective cannot improve, and may only get worse.

1b) If C_2=18, then ΔC_2=-2 which is within the allowable decrease of 5 units for the basis to be unchanged. Hence the optimum solution is unchanged but the objective is changed by $\Delta C_2 * X_2$ = -2*3 units, i.e. it drops to 84 (New Z^*=50*0 + **18***3 + 30*1 + 80*0 = 84).

If C_3=50, then ΔC_3=20 which is more than the allowable increase of 10 units for the basis to be unchanged. Hence the optimum solution is changed and nothing further can be said at this point.

If b_2=4, then Δb_2=-2 which is within the allowable decrease of 2.857 units for the basis to be unchanged. Hence the optimum basis is unchanged (i.e., X_2, X_3, S_1, S_4 continue to remain basic), but the **values** of these basic variables will change. While the new values cannot be found directly, the change in the objective may be found by using the shadow prices since the basis doesn't change. Since π_2 = -2.5, this means by definition of the shadow price that an increase of 1 unit improves the objective by -2.5 units. Thus an increase of -2 units improves the objective by (-2)*(-2.5)= 5 units, i.e., the new objective will be equal to 85. Note that this makes intuitive sense - the RHS of a ≥ constraint is being reduced so that it is being made easier to satisfy by expanding the feasible region to admit more points - thus the objective can only improve (be smaller for a minimization problem).

1c) From the computer output, b_3 can increase by up to 1.33 units or decrease by up to 4 units before the basis changes (i.e. as long as 6 ≤ new b_3 ≤ 11.33). Based on the shadow price of -7.5, in case of an increase the objective will improve by up to -7.5*1.33 = -10 units, i.e., **increase** by up to 10 units), and in case of a decrease it will improve by up to -7.5*-4=30 units, i.e., **decrease** by up to 30 units. Note that negative improvement implies increase and positive improvement implies decrease in a Min problem...)

1d) The optimum reduced cost value of 50 implies that (a) each 1 unit increase in X_4 (from its current value of 0) will cause Z to increase by 50 units, and (b) that the coefficient of X_4 would have to **decrease** by 50 units (i.e., drop to 30) before X_4 could become positive and enter the basis in an optimal solution.

2a) From the tableau, S_2 has a reduced cost of -2.5, so that if it is increased by 1 unit, the value of Z will **decrease** by -2.5*1, i.e., **increase** by 2.5 units.

2b) The substitution rates are interpreted as follows: for each 1 unit increase X_1, in order to maintain feasibility

- S_1 must be **decreased** by 137.5 units (subs. rate >0)
- X_2 must be **decreased** by 1.5 units (subs. rate >0)
- X_3 must be **increased** by 0.25 units (subs. rate <0)
- S_4 must be **decreased** by 3.75 units (subs. rate >0)

The leaving variable is determined from

- Min{50/137.5, 3/1.5, ∞, 5/3.75} = 50/137.5 corresponding to S_1.

Thus the maximum increase possible in X_1 is 50/137.5 units at which point S_1 will be equal to 0 and hence become nonbasic and leave the basis. [Note that at this point X_2 will be 3-1.5*(50/137.5), X_3 will be 1+0.25*(50/137.5) and S_4 will be 5-3.75(50/137.5)].

The **in**crease in Z at the next iteration = |reduced cost of X_1| * (increase in value of X_1)
= |reduced cost of X_1| * (Minimum ratio value) = 27.5*(50/137.5).
Thus new Z = 90 + 27.5*(50/137.5) = 100

QUESTION 3 *(WIVCO Computers)*

a) Here b_3=87 (rather than 90) and since Δb_3 = -3 is within the allowable decrease of 23.33 units for the basis not to change, we may use the shadow price of π_3=2.6 to infer that the profits will increase by 2.6*-3, i.e., new Z = 274 - 3*2.6 = 266.20
b) In this case the objective coefficient c_2 for x_2 now becomes 39.5*0.33 = 13.035 - a decrease of 0.165 units. This is not sufficient to change the basis (since it is less than the allowable decrease of 0.2 units), and the solution is thus unchanged. However, (new value of Z) = (old value of Z) + (-0.165*20) = 270.70
c) The shadow price associated with constraint 3 is π_3=2.6 so that Wivco should be willing to pay up to 2.60 more (i.e., 12.60) for each additional pound of raw material.
d) The shadow price for labor is π_2=0.2, i.e., Wivco should be willing to pay up to 20 cents more per hour of labor.

QUESTION 4 (*Cornco*)

Let P*i*= units of PS produced in month *i*
P*i*S = units of PS sold in month *i*
IP*i* = inventory of PT at end of month *i*
Q*i* = units of QT produced in month *i*
Q*i*S = units of QT sold in month *i*
IQ*i* = inventory of QT at end of month *i*
RM = pounds of raw material purchased.

Then the formulation is as follows:

```
 MAX     40 P1S + 60 P2S + 55 P3S + 35 Q1S + 40 Q2S + 44 Q3S - 3 RM
       - 10 IP1 - 10 IP2 - 10 IP3 - 10 IQ1 - 10 IQ2 - 10 IQ3
 SUBJECT TO
      1)   P1S <=   50
      2)   P2S <=   45
      3)   P3S <=   50
      4)   Q1S <=   43
      5)   Q2S <=   50
      6)   Q3S <=   40
      7)   3 P1 + 2 Q1 <=   1200
      8)   3 P2 + 2 Q2 <=   160
      9)   3 P3 + 2 Q3 <=   190
      10)  2 P1 + 2 Q1 <=   2140
      11)  2 P2 + 2 Q2 <=   150
      12)  2 P3 + 2 Q3 <=   110
      13)  P1S + IP1 - P1 =    10
      14)  P2S - IP1 + IP2 - P2 =    0
      15)  P3S - IP2 + IP3 - P3 =    0
      16) Q1S + IQ1 - Q1 =    5
      17)  Q2S - IQ1 + IQ2 - Q2 =    0
      18)  Q3S - IQ2 + IQ3 - Q3 =    0
      19) - RM + 4 P1 + 3 Q1 + 4 P2 + 3 Q2 + 4 P3 + 3 Q3 =    0
      20)  RM <=   710
  END
```

The corresponding output from LINDO is as follows:

LP OPTIMUM FOUND AT STEP 15

OBJECTIVE FUNCTION VALUE

Obj) 7705.000

VARIABLE	VALUE	REDUCED COST
P1S	22.750000	.000000
P2S	45.000000	.000000
P3S	50.000000	.000000
Q1S	43.000000	.000000
Q2S	50.000000	.000000
Q3S	5.000000	.000000
RM	710.000000	.000000
IP1	25.000000	.000000
IP2	.000000	6.000000
IP3	.000000	64.000000
IQ1	.000000	3.333333
IQ2	.000000	2.666667
IQ3	.000000	54.000000
P1	37.750000	.000000
Q1	38.000000	.000000
P2	20.000000	.000000
Q2	50.000000	.000000
P3	50.000000	.000000
Q3	5.000000	.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
1)	27.250000	.000000
2)	.000000	10.000000
3)	.000000	1.000000
4)	.000000	5.000000
5)	.000000	3.333333
6)	35.000000	.000000
7)	1010.750000	.000000
8)	.000000	3.333333
9)	30.000000	.000000
10)	1988.500000	.000000
11)	10.000000	.000000
12)	.000000	7.000000
13)	.000000	40.000000
14)	.000000	50.000000
15)	.000000	54.000000
16)	.000000	30.000000
17)	.000000	36.666670
18)	.000000	44.000000
19)	.000000	10.000000
20)	.000000	7.000000

NO. ITERATIONS= 15

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
P1S	40.000000	4.000000	3.555557
P2S	60.000000	INFINITY	10.000000
P3S	55.000000	INFINITY	1.000000
Q1S	35.000000	INFINITY	5.000000
Q2S	40.000000	INFINITY	3.333333
Q3S	44.000000	1.000000	14.000000
RM	-3.000000	INFINITY	7.000000
IP1	-10.000000	4.000002	4.999998
IP2	-10.000000	6.000000	INFINITY
IP3	-10.000000	64.000000	INFINITY
IQ1	-10.000000	3.333332	INFINITY
IQ2	-10.000000	2.666668	INFINITY
IQ3	-10.000000	54.000000	INFINITY
P1	.000000	4.000000	4.999998
Q1	.000000	3.333332	5.000000
P2	.000000	4.999998	4.000002
Q2	.000000	2.666668	3.333332
P3	.000000	64.000000	1.000000
Q3	.000000	1.000000	14.000000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
1	50.000000	INFINITY	27.250000
2	45.000000	22.750000	25.000000
3	50.000000	5.000000	35.000000
4	43.000000	30.333330	36.333330
5	50.000000	15.000000	36.333330
6	40.000000	INFINITY	35.000000
7	1200.000000	INFINITY	1010.750000
8	160.000000	15.000000	60.000000
9	190.000000	INFINITY	30.000000
10	2140.000000	INFINITY	1988.500000
11	150.000000	INFINITY	10.000000
12	110.000000	30.000000	10.000000
13	10.000000	27.250000	22.750000
14	.000000	25.000000	22.750000
15	.000000	35.000000	5.000000
16	5.000000	36.333330	30.333330
17	.000000	36.333330	15.000000
18	.000000	35.000000	5.000000
19	.000000	109.000000	91.000000
20	710.000000	109.000000	91.000000

a) If inventory costs are $11 for PS in month 1 then the coefficient for IP1 decreases by 1, and since the basis is unchanged (within allowable decrease), profits go down by IP1*1 = 25*1 = $25

b) RHS for the constraint (Row 7) drops from 1200 to 210, i.e., by 990 units, which is less than the allowable decrease for the basis to remain unchanged (= 1010.75). Thus basis is unchanged. The slack variable associated with this constraint (row 8) continues to be positive, the shadow price associated with the constraint is 0, and the change in the profit = 0. The solution is thus unchanged.

c) The RHS for the constraint (Row 12) is now 109, i.e., it drops by 1 unit which is within the allowable decrease of 10 for the basis to be unchanged. Then since the shadow price for the constraint is 7, the profit increases by 7*-1, i.e., drops by 7 units to 7705-7 = $7698.

d) Line 1 time constraint is Row 8 with shadow price of 3.33 and since a 1 unit increase will not change the basis (allowable increase = 15), so that profits will rise by $3.33 for each extra hour on Line 1. So we would be willing to pay up to $3.33 for an extra hour.

e) The shadow price for raw material (Row 20) is 7, so using the same argument as for Part (d) above, the answer is $7.

f) Since there is a positive slack in this constraint (Row 9), there is no need to buy extra time on Line 1 in month 3 - the shadow price is 0 and the profits will not increase for an extra hour. So the answer is 0.

g) If PS sells for $50 in month 2 then the coefficient for P2S drops by 10 units - this is within the allowable decrease of 10 so that the basis is unchanged and thus the profits drop by 10*P2S = 10*45 to a value of 7705-450= $7255.

h) If QT sells for $50 in month 3 then the coefficient for Q3S rises by 6 units which is **more** than the allowable increase of 1. Thus the basis changes and nothing can be said at this point about the new optimum solution or profits.

i) The constraint for QT demand in month 2 is Row 5, which has a shadow price of 3.33 and the allowable increase in the RHS for the basis not to change is 15 units. So increasing demand by 5 units will leave the basis unchanged and increase overall profits by 3.33*5 - 20 = -3.35. Thus the advertising should not be done (it should be done only if the cost is 20-3.35=$16.65 or lower).