Question 1:
Question 1, page 398 of the text (five employees & four jobs...)

After adding a dummy job column and “M” to denote the cost of disallowed assignments we obtain the following cost matrix on the left:

\[
\begin{array}{cccccc}
22 & 18 & 30 & 18 & 0 & \\
18 & M & 27 & 22 & 0 & \\
26 & 20 & 28 & 28 & 0 & \\
16 & 22 & M & 14 & 0 & \\
21 & M & 25 & 28 & 0 & \\
\end{array}
\]

Since the matrix is already row reduced, we column reduce it to obtain the second matrix (note that one could use M-18 in column 2 and M-25 in column 3 but assuming that M is “very” large, one could just as well use “M” instead). Since all zeroes can be covered using four lines as shown, we're not done. The smallest uncovered cost is 2 and subtracting this from other uncrossed entries and adding it to all twice crossed entries we obtain the third matrix. Five lines are needed to cover all zeroes here.

We're done and the optimal assignment can be extracted from the table with candidate assignments where the 0’s are located (shown below to the left): Person 1 does job 2, Person 2 does job 1, Person 3 is idle (job 5 is a dummy job), Person 4 does job 4, and Person 5 does job 3.

Total time = 18 + 18 + 0 + 14 + 25 = 75 hours.

Question 2.
Question 28, page 411 of the text (professors...)

Since each professor must teach two sections we make an extra “copy” of each professor, for a total of 6 agents. Then to convert the problem to a minimization, we multiply all the rankings by -1. The “cost” matrix is given by the following

\[
\begin{array}{ccccccc}
\text{Section Professor} & \text{9 AM} & \text{10 AM} & \text{11 AM} & \text{1 PM} & \text{2 PM} & \text{3 PM} \\
1 & -8 & -7 & -6 & -5 & -7 & -6 \\
1' & -8 & -7 & -6 & -5 & -7 & -6 \\
2 & -9 & -9 & -8 & -8 & -4 & -4 \\
2' & -9 & -9 & -8 & -8 & -4 & -4 \\
3 & -7 & -6 & -9 & -6 & -9 & -9 \\
3' & -7 & -6 & -9 & -6 & -9 & -9 \\
\end{array}
\]

Row reduction followed by column reduction yield the first and second matrices below:
Since the second matrix can be covered with 5 lines as shown above, we proceed with the next step (smallest uncovered entry = 1).

This yields the optimal tableau and the table with candidate assignments where the 0’s are located (shown below to the left) assignments shown: Professor 1 teaches 9 AM and 2 PM sections, Professor 2 teaches 10 AM and 1 PM sections, and Professor 3 teaches 11 AM and 3 PM sections.

Maximum total ranking = 8+7+9+8+9+9 = 50.

**Question 3.**

Question 1, at the end of Section 9.2 on page 502 of the text (Coach Night)

Define \( x_j = 1 \) if player \( j \) starts; \( j=1,2,3,4,5,6,7 \)
0 otherwise.

Then the appropriate IP is as follows:

Maximize \( Z = 3x_1 + 2x_2 + 2x_3 + x_4 + 3x_5 + 3x_6 + x_7 \)

Subject to
1. \( x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 5 \) (five on a team)
   \( x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \geq 4 \) (or =4)
   \( x_3 + x_4 + x_5 + x_6 + x_7 \geq 2 \) (forwards)
   \( x_2 + x_4 + x_6 \geq 1 \) (center)

2. \( (3x_1 + 2x_2 + 2x_3 + x_4 + 3x_5 + 3x_6 + 3x_7) \geq 2 \), i.e.,
   \( 3x_1 + 2x_2 + 2x_3 + x_4 + 3x_5 + 3x_6 + 3x_7 \geq 10 \) (avg. ball handling)
   \( (3x_1 + x_2 + 3x_3 + x_4 + 3x_5 + x_6 + 2x_7) \geq 2 \), i.e.,
   \( 3x_1 + x_2 + 3x_3 + x_4 + 3x_5 + x_6 + 2x_7 \geq 10 \) (avg. shooting)
   \( (x_1 + 3x_2 + 2x_3 + x_4 + x_5 + 2x_6 + 2x_7) \geq 2 \), i.e.
   \( x_1 + 3x_2 + 2x_3 + x_4 + x_5 + 2x_6 + 2x_7 \geq 10 \) (avg. rebounding)

5. \( x_2 + x_7 \geq 1 \) (either 2 or 3 must start)

All \( x_{ij} = 0 \) or 1
**Question 4.**

Question 3, pages 502-503 of the text (manufacturer...)

Define \( x_1 \) = units of product 1 produced,
\( x_2 = \) units of product 2 produced,
\( y_1 = 1 \) if any product 1 is produced; 0 otherwise.
\( y_2 = 1 \) if any product 2 is produced; 0 otherwise.

Then the problem formulated is as follows (note that this is a fixed-charge problem where fixed charges are incurred for each product whenever any amount of that product is produced):

Maximize \( Z = 2x_1 + 5x_2 - 10y_1 - 20y_2 \)

st
1. \( 3x_1 + 6x_2 \leq 120 \) (raw material availability)
2. \( x_1 \leq My_1 \) (if \( y_1 = 0 \) then \( x_1 \) must be = 0, if \( y_1 = 1 \) then \( x_1 \) is limited only by constraint 1)
3. \( x_2 \leq My_2 \) (if \( y_2 = 0 \) then \( x_2 \) must be = 0, if \( y_2 = 1 \) then \( x_2 \) is limited only by constraint 1)

\( x_1, x_2 \geq 0; \ y_1, y_2 = 0 \ or \ 1 \)

**Note:** While any "large" value for \( M \) would be okay, it would suffice to use 120/3=40 for \( M \) in (2) and 120/6=20 for \( M \) in (3).

**Question 5.**

Question 13, page 503 of the text (Glueco...)

Define \( y_j = 1 \) if production line \( j \) is used and 0 otherwise, \( j=1,2 \)
\( x_j = \) No. of workers assigned to line \( j, j=1,2 \)

Then the formulation would be to minimize

Minimize \( Z = 500x_1 + 900x_2 + 1000y_j + 2000y_2 \) (weekly costs)

st
1. \( 20x_1 + 50x_2 \geq 120 \)
2. \( 30x_1 + 35x_2 \geq 150 \)
3. \( 40x_1 + 45x_2 \geq 200 \) (production requirements for the 3 glue types)
4. \( x_1 \leq 7y_1 \)
5. \( x_2 \leq 7y_2 \) (if line \( j \) is not used \( x_j = 0 \), and if it is used then \( x_j \leq 7 \))

\( x_j, x_2 \geq 0 \) and integer, \( y_j, y_2 = 0 \ or \ 1 \).