A Typical Transportation Problem

The Industrial Engineering Department at a large corporation has been assigned the task of planning the distribution of the monthly production at its three main plants to its four major warehouses around the country. The following demand / supply and cost data are available:

Plant i	Warehouse j				Supply
	j=1	j=2	j=3	j=4	S_j
i=1	$12	$13	$4	$6	*500*
i=2	$6	$4	$10	$11	*700*
i=3	$10	$9	$12	$14	*800*
Demand D_j	*400*	*900*	*200*	*500*	*2000*

QUESTION: What is the minimum cost distribution plan so that all demand is satisfied ?

Formulation of the Corresponding Transportation Model

ANSWER: Solve the following LP

Define X_{ij} = Amount shipped from source (plant) i to destination (warehouse) j

Min $12X_{11}+ 13X_{12}+ 4X_{13}+ 6X_{14}+ 6X_{21}+ 4X_{22}+ 10X_{23}+ 11X_{24}+ 10X_{31}+ 9X_{32}+ 12X_{33}+ 14X_{34}$

st

$$X_{11}+X_{12}+X_{13}+X_{14} \leq 500$$

$$X_{21}+X_{22}+X_{23}+X_{24} \leq 700$$

$$X_{31}+X_{32}+X_{33}+X_{34} \leq 800$$

$$X_{11}+X_{21}+X_{31} \geq 400$$

$$X_{12}+X_{22}+X_{32} \geq 900$$

$$X_{13}+X_{23}+X_{33} \geq 200$$

$$X_{14}+X_{24}+X_{34} \geq 500$$

all $X_{ij} \geq 0$

Balanced Transportation Problem

In general, it is not a requirement that total supply must equal total demand, i.e., that $\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j$

- $\sum_i S_i > \sum_j D_j$ means supply exceeds demand, so a total of $(\sum_i S_i) - (\sum_j D_j)$ units will remain at one or more of the sources
- $\sum_i S_i < \sum_j D_j$ means demand exceeds supply, so a total of $(\sum_j D_j) - (\sum_i S_i)$ units if demand across all of the destinations will not be met

A BALANCED transportation problem is one where total supply is equal to total demand, i.e., $\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j$

Such a formulation often makes sense when there is a cost associated with not meeting demand or having inventory at the sources

Obtaining a Balanced Formulation

Case 1: $\sum_i S_i > \sum_j D_j$

- Define a dummy destination (n+1) with demand $D_{n+1} = (\sum_i S_i) - (\sum_j D_j)$ to absorb the excess supply
- Define transportation cost $c_{i,n+1}$ as the cost of retaining 1 unit at source i

Case 2: $\sum_i S_i < \sum_j D_j$

- Define a dummy source (m+1) with supply $S_{m+1} = (\sum_j D_j) - (\sum_i S_i)$ to supply the excess demand
- Define transportation cost $c_{m+1,j}$ as the cost of not meeting 1 unit of demand at destination j

Note that shipments out of a dummy source or into a dummy destination do not actually happen (because the dummy does not actually exist)! So we associate the appropriate cost with these.

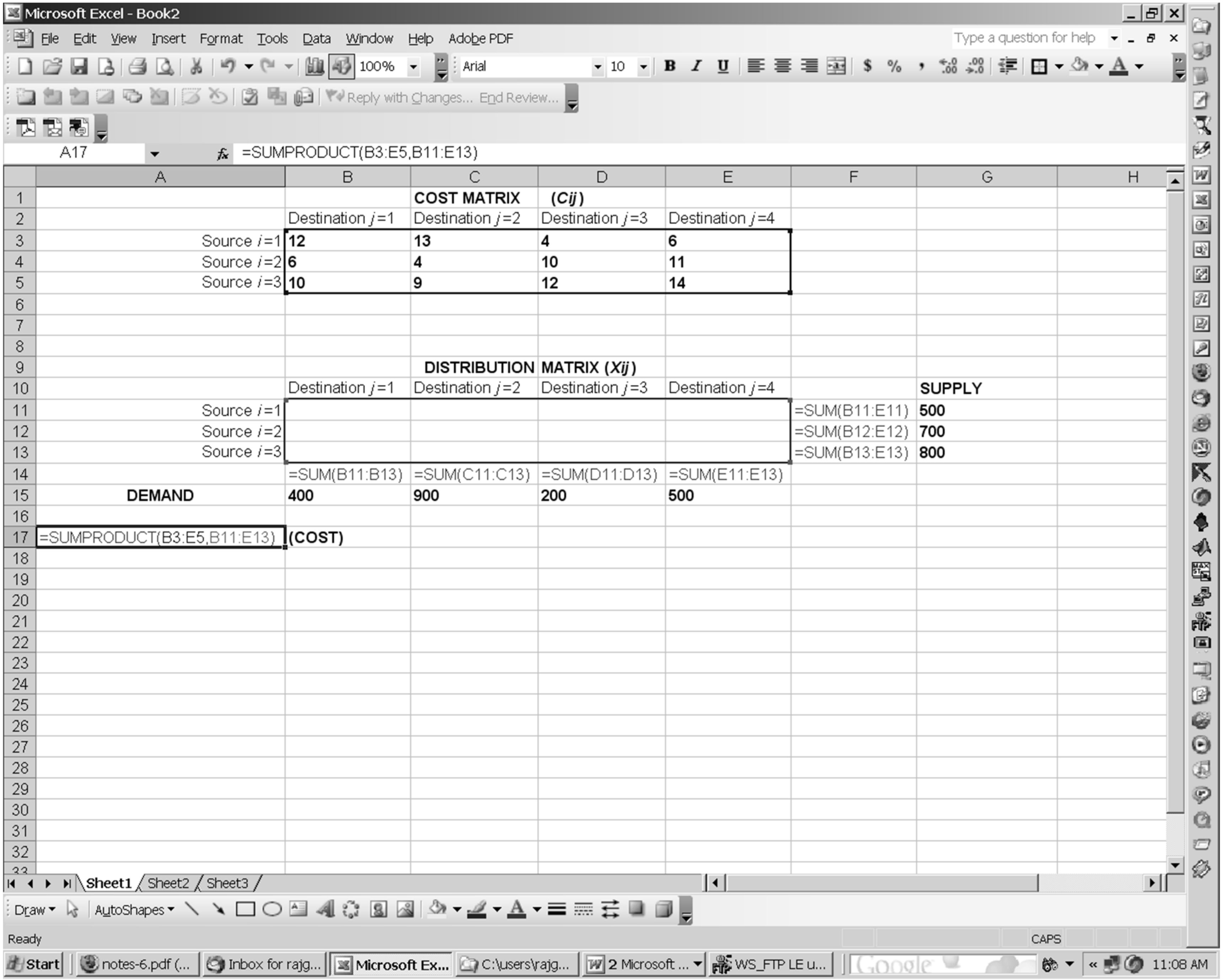
Transportation Problem

The transportation problem has some interesting properties:

1. If the supplies (S_i) and demands (D_j) are integers, then it is guaranteed that the optimal solution to the LP formulation will also be integer valued

2. In a balanced transportation problem, there is always one redundant constraint; so you can arbitrarily drop one of the supply or demand constraints before solving it

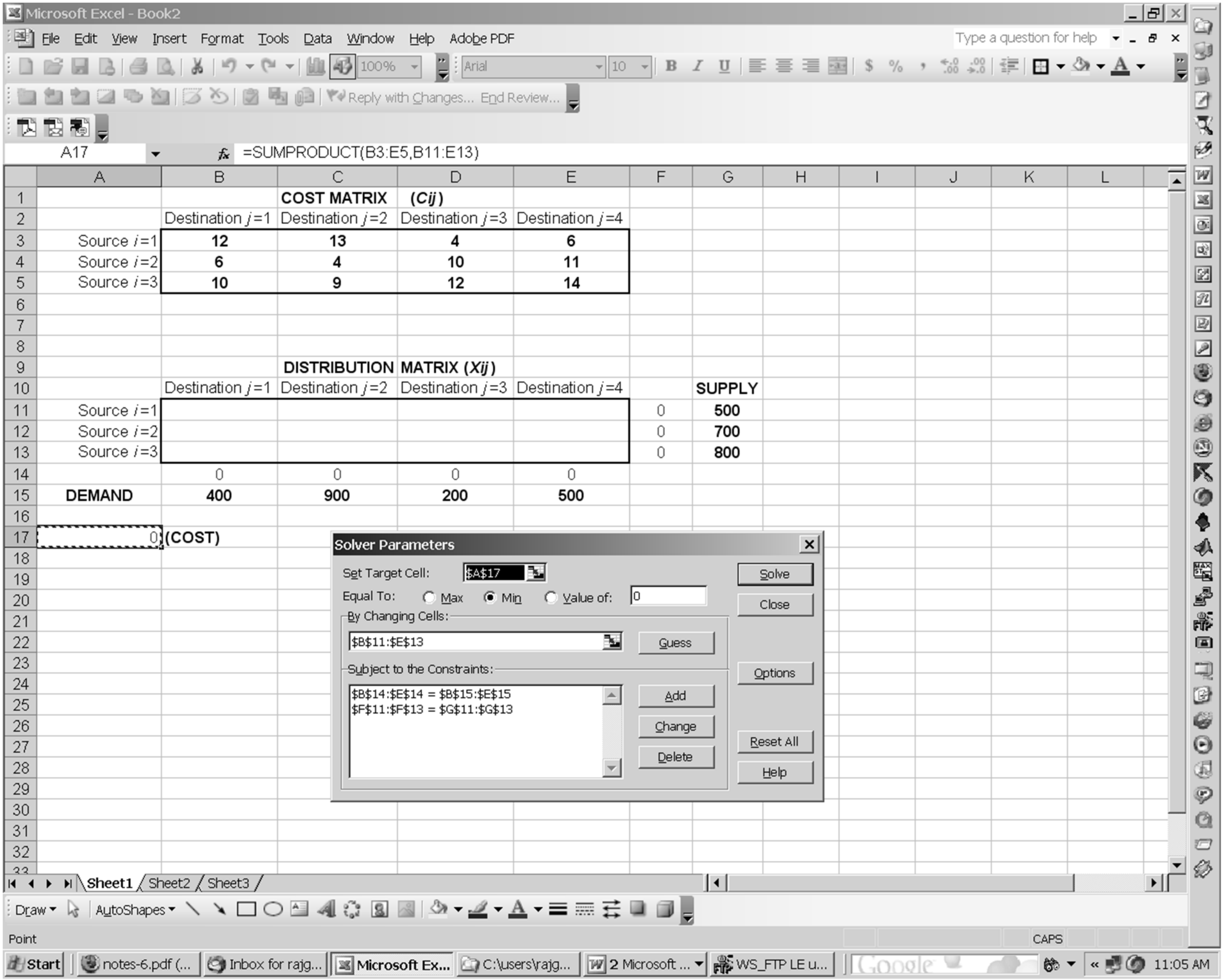
3. There is a very efficient version of the Simplex method called the Transportation Simplex method that only uses addition and subtraction operations to solve the transportation problem

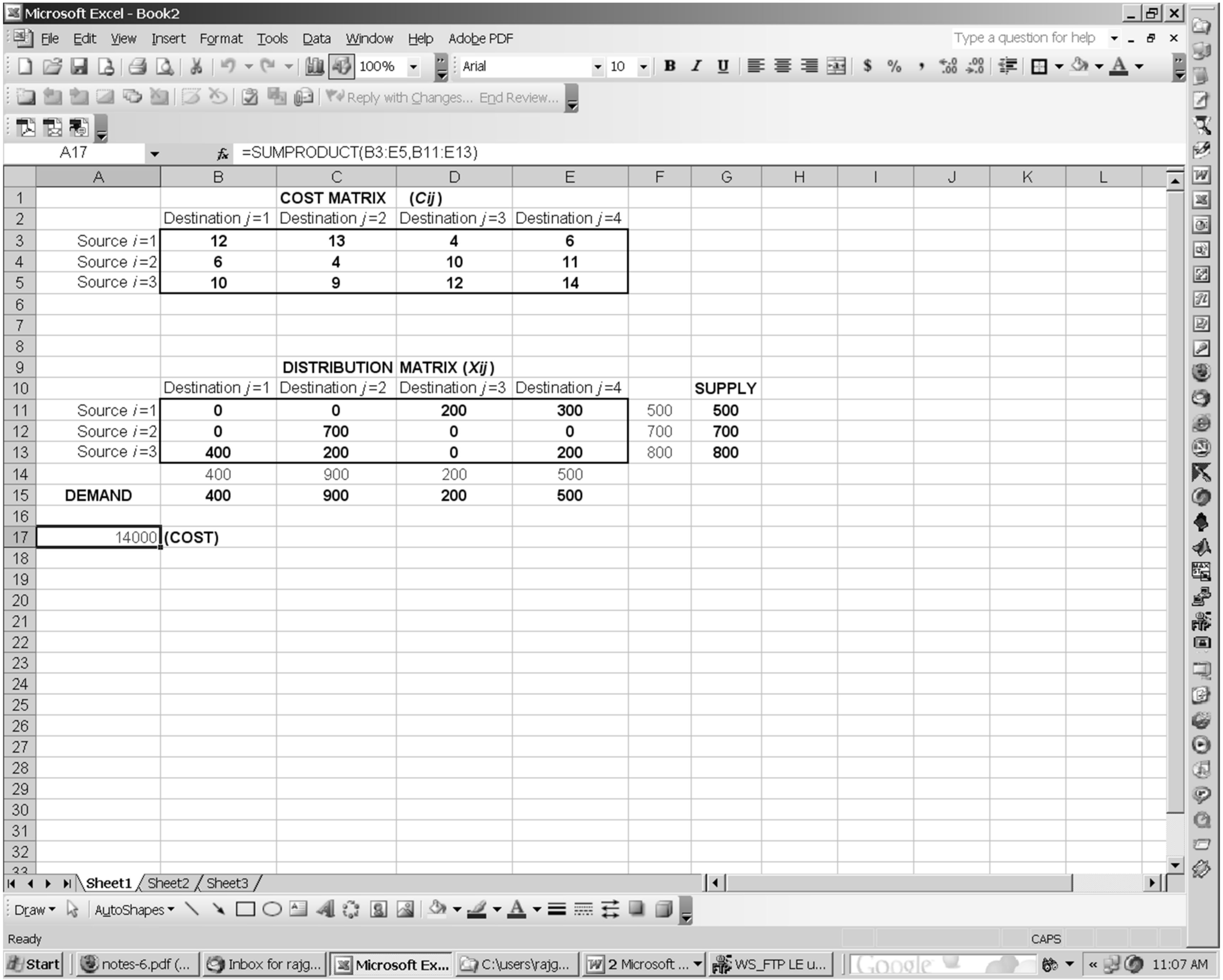


Microsoft Excel - Book2

A17 =SUMPRODUCT(B3:E5,B11:E13)

	A	B	C	D	E	F	G
1			COST MATRIX	(*Cij*)			
2		Destination *j*=1	Destination *j*=2	Destination *j*=3	Destination *j*=4		
3	Source *i*=1	**12**	**13**	**4**	**6**		
4	Source *i*=2	**6**	**4**	**10**	**11**		
5	Source *i*=3	**10**	**9**	**12**	**14**		
6							
7							
8							
9			**DISTRIBUTION**	**MATRIX (*Xij*)**			
10		Destination *j*=1	Destination *j*=2	Destination *j*=3	Destination *j*=4		**SUPPLY**
11	Source *i*=1					=SUM(B11:E11)	**500**
12	Source *i*=2					=SUM(B12:E12)	**700**
13	Source *i*=3					=SUM(B13:E13)	**800**
14		=SUM(B11:B13)	=SUM(C11:C13)	=SUM(D11:D13)	=SUM(E11:E13)		
15	**DEMAND**	**400**	**900**	**200**	**500**		
16							
17	=SUMPRODUCT(B3:E5,B11:E13)	**(COST)**					

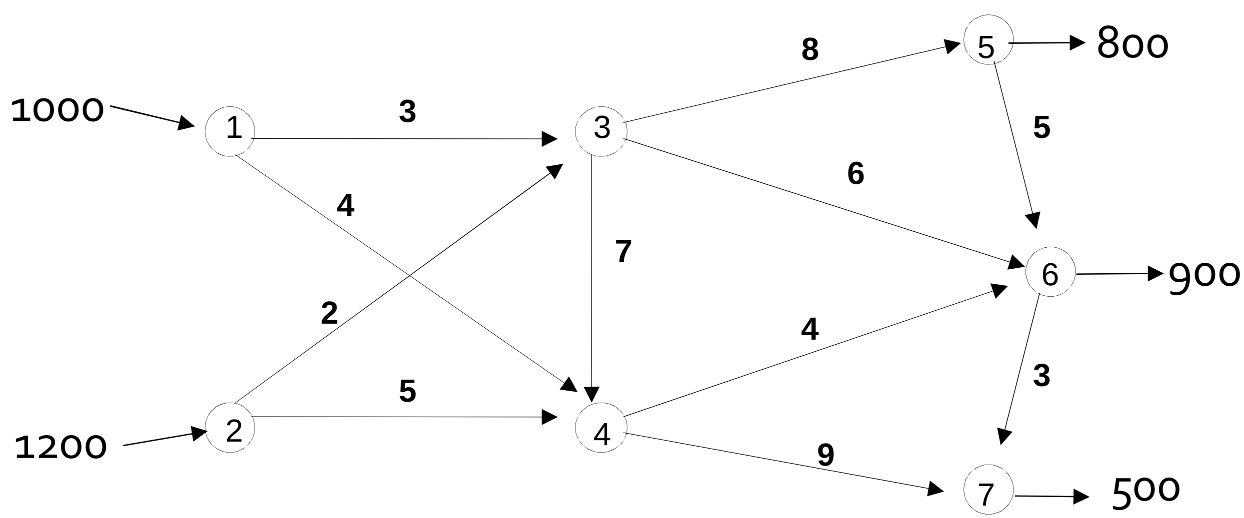

Microsoft Excel - Book2
A17
=SUMPRODUCT(B3:E5,B11:E13)
COST MATRIX (Cij)
Destination j=1 Destination j=2 Destination j=3 Destination j=4
Source i=1 12 13 4 6
Source i=2 6 4 10 11
Source i=3 10 9 12 14
DISTRIBUTION MATRIX (Xij)
Destination j=1 Destination j=2 Destination j=3 Destination j=4 SUPPLY
Source i=1 0 500
Source i=2 0 700
Source i=3 0 800
0 0 0 0
DEMAND 400 900 200 500
0 (COST)
Solver Parameters
Set Target Cell: $A$17
Equal To: Max Min Value of: 0
By Changing Cells:
$B$11:$E$13
Subject to the Constraints:
$B$14:$E$14 = $B$15:$E$15
$F$11:$F$13 = $G$11:$G$13
Solve
Close
Guess
Options
Add
Change
Delete
Reset All
Help
Sheet1 Sheet2 Sheet3



	A	B	C	D	E	F	G
1			COST MATRIX	(Cij)			
2		Destination j=1	Destination j=2	Destination j=3	Destination j=4		
3	Source i=1	12	13	4	6		
4	Source i=2	6	4	10	11		
5	Source i=3	10	9	12	14		
6							
7							
8							
9			DISTRIBUTION	MATRIX (Xij)			
10		Destination j=1	Destination j=2	Destination j=3	Destination j=4		SUPPLY
11	Source i=1	0	0	200	300	500	500
12	Source i=2	0	700	0	0	700	700
13	Source i=3	400	200	0	200	800	800
14		400	900	200	500		
15	DEMAND	400	900	200	500		
16							
17	14000	(COST)					

A Transshipment Problem

Consider an automobile company with two plants located at nodes 1 and 2, with supply amounts of 1000 and 1200 respectively. Cars are shipped to dealers denoted by nodes 5, 6 and 7 through distribution centers denoted by 4 and 5. The demands at the three dealers are 800, 900 and 500 respectively. The number above each arc represents the per unit shipping cost.



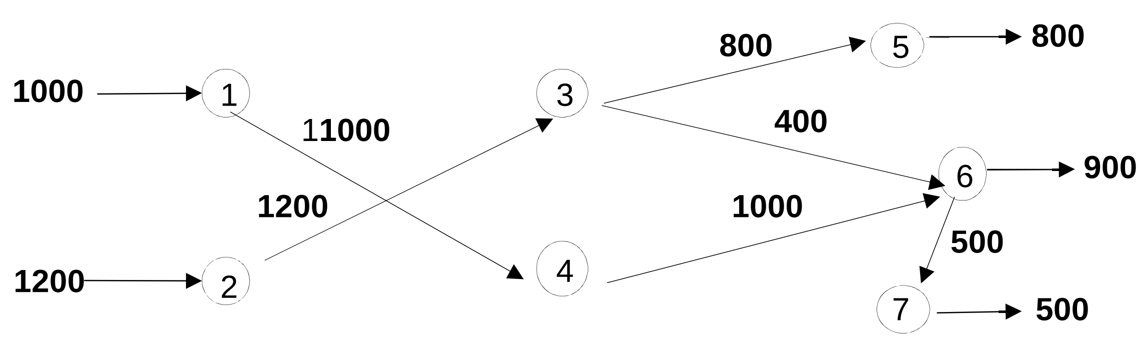
What is the best way of fulfilling demand at the dealers?

Formulating the *Transshipment* Problem as a *Transportation* Problem

	3	*4*	*5*	*6*	*7*	
1	3	4	M	M	M	**1000**
2	2	5	M	M	M	**1200**
3	0	7	8	6	M	**2200**
4	M	0	M	4	9	**2200**
5	M	M	0	5	M	**2200**
6	M	M	M	0	3	**2200**
	2200	**2200**	**3000**	**3100**	**500**	

The Optimal Solution

	3	4	5	6	7	
1	3	4 1000	M	M	M	**1000**
2	2 1200	5	M	M	M	**1200**
3	0 1000	7	8 800	6 400	M	**2200**
4	M	0 1200	M	4 1000	9	**2200**
5	M	M	0 2200	5	M	**2200**
6	M	M	M	0 1700	3 500	**2200**
	2200	**2200**	**3000**	**3100**	**500**	



The Assignment Problem

As part of his annual audit, the president of a large corporation has decided to have each of his four corporate vice-presidents visit and audit one of the corporation's four assembly plants during a two-week period in June. These plants are located in Leipzig, Germany; Nancy, France; Liege, Belgium and Tilburg, the Netherlands.

There are numerous advantages and disadvantages to various assignments; among the issues to consider are:

1. Matching the vice-presidents' areas of expertise with the importance of specific problem areas in a plant.
2. The time the audit will require and the other demands on each vice-president during the two-week interval.
3. Matching the language ability of a vice-president with the dominant language used in the plant.

Attempting to keep all these factors in mind, the goal is to arrive at a good assignment of vice-presidents to plants. The president has started by estimating the cost of sending each vice-president to each plant; the data are shown in Table 1.

	Plant			
V.P.	Leipzig (1)	Nancy (2)	Liege (3)	Tilburg (4)
Finance (F)	24	10	21	11
Marketing (M)	14	22	10	15
Operations (O)	15	17	20	19
Personnel (P)	11	19	14	13

What is the optimal assignment?

The Assignment Problem

Note that the Problem can be represented as the following transportation cost-requirements matrix:

Plant (*j*) / V.P. (*i*)	Leipzig (1)	Nancy (2)	Liege (3)	Tilburg (4)	No. of V.P.'s available
Finance (F)	*24*	*10*	*21*	*11*	1
Marketing (M)	*14*	*22*	*10*	*15*	1
Operations (O)	*15*	*17*	*20*	*19*	1
Personnel (P)	*11*	*19*	*14*	*13*	1
No. of V.P.'s required	1	1	1	1	4

The HUNGARIAN Algorithm for the Assignment Problem

Consider the $n \times n$ assignment cost matrix **A**. Define:

- Row Reduction Operation: Subtract the smallest entry in a row from every entry in that row.
- Column Reduction Operation: Subtract the smallest entry in a column from every entry in that column.
- Fully Reduced Matrix: One where the **minimum** number of horizontal and/or vertical lines required to cover all zero entries is equal to the order of the matrix (n).

STEP 0: First Row Reduce A; then Column Reduce the resulting matrix.

STEP 1: Is the current matrix fully reduced? If yes go to Step 3; else go to Step 2.

STEP 2: Identify the **smallest unlined cell value** m. Subtract m from every entry that is **unlined**, and add m to every entry with **two** lines through it. Go to Step 1 with the new matrix.

STEP 3: Extract the optimal assignment.