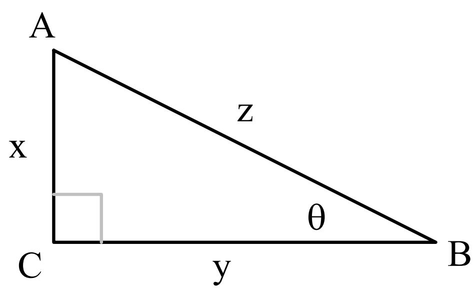
MA0221	Trigonometry Basics	NAME

Introduction

Trigonometry means to "measure triangles" so we define sine, cosine and tangent on "nice" triangles, i.e. right triangles,



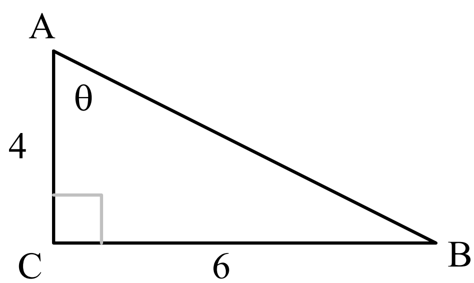
$$\sin\theta = \frac{x}{z} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{y}{z} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{x}{y} = \frac{\text{opposite}}{\text{adjacent}}$$

where θ must be strictly between 0° and 90°, $0° < \theta < 90°$. Since we are dealing with right triangles we know the Pythagorean Theorem, $x^2 + y^2 = z^2$, in the above diagram.

Exercise 1: Consider the following right triangle. Use the Pythagorean Theorem to determine the length AB. Using the ratio of sides determine sinθ, cosθ and tanθ.



AB = _____

sinθ = _____

cosθ = _____

tanθ = _____

Exercise 2: If $\sin\theta = \frac{5}{13}$ then determine cosθ and tanθ. (Recall, that $0° < \theta < 90°$.)

cosθ = _____ tanθ = _____

Basic Trigonometric Identities

Recall, $\csc\theta = \frac{1}{\sin\theta} = \frac{\text{hypotenuse}}{\text{opposite}}$, $\sec\theta = \frac{1}{\cos\theta} = \frac{\text{hypotenuse}}{\text{adjacent}}$ and $\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta} = \frac{\text{adjacent}}{\text{opposite}}$.

Exercise 3: Use your background or consult a resource (textbook, web, etc.) to complete the following identities.

$\sin^2\theta + \cos^2\theta =$ ______ (*)

$\tan^2\theta + 1 =$ ______ (**)

$1 + \cot^2\theta =$ ______ (***)

Exercise 4: What do you divide formula (*) by to obtain formula (**)?

Additional Identities

Use your resources to complete the following identities.

$\sin(\alpha + \beta) = \sin\alpha$ _________ $+ \cos\alpha$ _________

$\sin(\alpha - \beta) =$ ______________________________

$\cos(\alpha + \beta) = \cos\alpha \cos\beta -$ __________________

$\cos(\alpha - \beta) =$ ______________________________

$\sin 2\theta =$ _______________

$\cos 2\theta = \cos^2\theta -$ __________

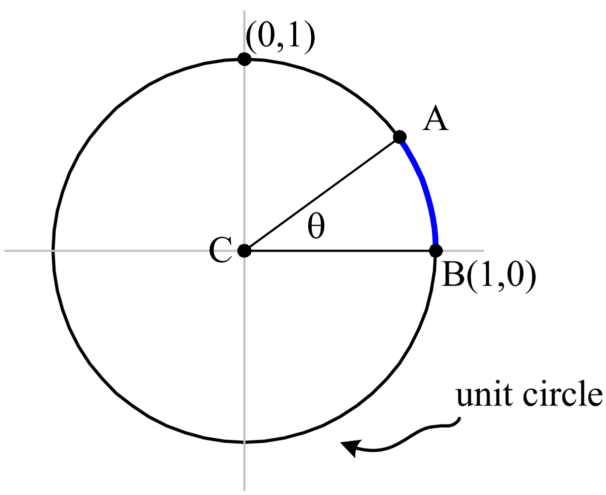
$= 1 - 2$___________

$= 2$___________ $- 1$

Notice that the identities for $\sin 2\theta$ and $\cos 2\theta$ come from letting $\alpha = \theta$ and $\beta = \theta$ and possibly using $\sin^2\theta + \cos^2\theta = 1$ in the identities for $\sin(\alpha + \beta)$ and $\cos(\alpha + \beta)$.

Trigonometric Functions

Ultimately, we want to think of sine, cosine and tangent as functions not merely as the ratio of sides of a right triangle. Thus, we want the domain to be $(-\infty,\infty)$ and not just $(0, \pi/2)$ as they are initially defined. First we want to recall radian measure.



Using the unit circle we determine the real number length of arc(AB), and define it to be the radian measure associated with the degree measure θ. So radians are real numbers while degrees are … well … degrees. We typically use radian measure in all applications because radians are real numbers.

Since the circumference of the unit circle is $C = 2\pi r = 2\pi(1) = 2\pi$ and 360° is associated with rotating the whole way around, the relationship between radians and degrees is $2\pi \leftrightarrow 360°$. Or divide through by 2 and you have $\pi \leftrightarrow 180°$.

Exercise 5: Determine the radian measure for 30° and 60°. Determine the degree measure for $\frac{5\pi}{4}$.

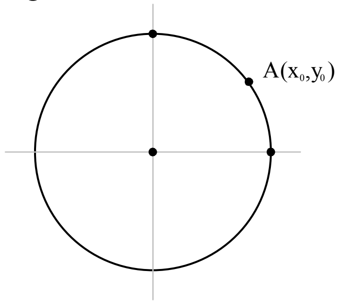
$30° \leftrightarrow$ __________

$60° \leftrightarrow$ __________

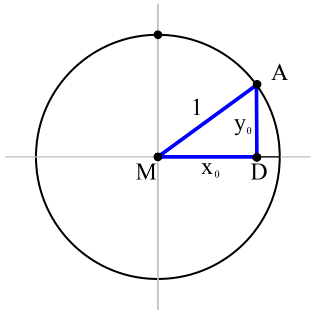
$\frac{5\pi}{4} \leftrightarrow$ __________

We now extend the definition of the sine and cosine function beyond $0° < \theta < 90°$ so that we can think of them as functions on the entire real number line $(-\infty,\infty)$. Notice that I will interchangeably use degrees and radians so that $0° < \theta < 90°$ and $0 < \theta < \frac{\pi}{2}$ designate the same things, but we still use radians in our computations.

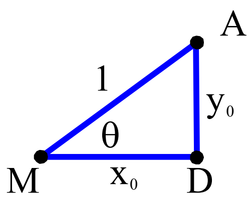
Again, consider the unit circle centered at the origin and place a point A on the circle (1st quadrant).



Assume that the point A has the coordinates (x_0,y_0).



Using the point A and the x-axis, we can see there is always an associated right triangle with point A, where the side lengths of the triangle are x_0, y_0 and 1 (the radius of the circle). Using this right triangle in the first quadrant we can incorporate sine and cosine.



Namely, $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{y_0}{1} = y_0$ and $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{x_0}{1} = x_0$.

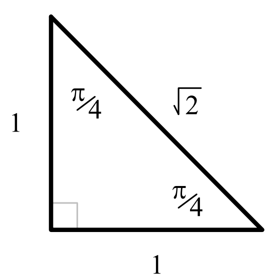
Thus, we extend the definition of sine and cosine beyond $0° < \theta < 90°$ by defining them to be the associated coordinate from point A on the circle. That means that $\sin\theta = y_0$ and $\cos\theta = x_0$.

Exercise 6: Determine the quadrants where sine is positive. Determine the quadrants where cosine is positive. What trigonometric functions are positive in the 3rd quadrant?

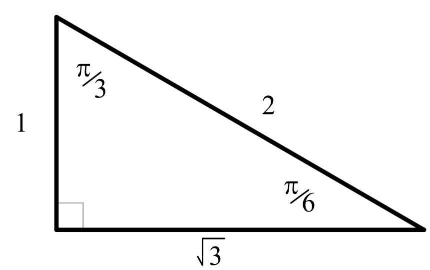
Trigonometric Function	Quadrants
sinθ	
cosθ	
	III

Important Triangles

There are two important triangles that are helpful to remember.



45-45-90



30-60-90

Applying Our Trigonometric Knowledge

Example: Solve $\sin^2\theta - \cos^2\theta = 0$ for θ, $0 \le \theta \le 2\pi$.

$$0 = \sin^2\theta - \cos^2\theta$$
$$= (\sin\theta - \cos\theta)(\sin\theta + \cos\theta) \qquad \text{(difference of two squares, } a^2 - b^2 = (a-b)(a+b)\text{)}$$

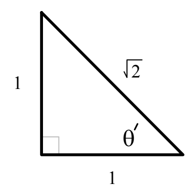
So either

$$\sin\theta - \cos\theta = 0 \qquad \text{or} \qquad \sin\theta + \cos\theta = 0$$
$$\sin\theta = \cos\theta \qquad\qquad \sin\theta = -\cos\theta\cdot$$
$$\frac{\sin\theta}{\cos\theta} = 1 \qquad\qquad \frac{\sin\theta}{\cos\theta} = -1$$
$$\tan\theta = 1 \qquad\qquad \tan\theta = -1$$

Using reference angles we have

$\tan\theta' = 1$ and $\tan\theta' = |-1| = 1$.

In both cases the reference angle refers to the following important triangle.



So the reference angle is $\frac{\pi}{4}$ for both cases.

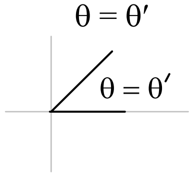
For $\tan\theta = 1$ tangent is positive (1) so θ must be in the 1st or the 3rd quadrant.

and for

$\tan\theta = -1$ tangent is negative (-1) so θ must be in the 2nd or the 4th quadrant.

For the 1st quadrant

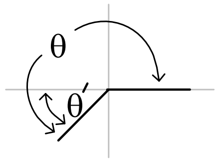
$\theta = \theta'$



so $\theta = \frac{\pi}{4}$.

For the 3rd quadrant

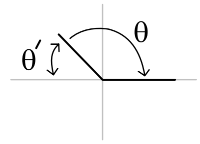
$\theta = \pi + \theta'$



so $\theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$.

For the 2nd quadrant

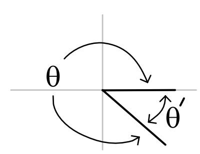
$\theta = \pi - \theta'$



so $\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$.

For the 4th quadrant

$\theta = 2\pi - \theta'$



so $\theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$.

Thus, all θ, $0 \le \theta \le 2\pi$, that solve $\sin^2\theta - \cos^2\theta = 0$ are $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$ and $\frac{7\pi}{4}$.

Exercise 7: Solve $\left(2\sin\theta + \sqrt{3}\right)\left(2\cos\theta - 1\right) = 0$ for θ, $0 \le \theta \le 2\pi$.