Department of Mathematics

University of Pittsburgh

MATH 2371

Final, Spring 2016

Instructor: Kiumars Kaveh

Last Name: **Student Number:**

First Name:

TIME ALLOWED: 50 MINUTES. TOTAL: 50
NO AIDS ALLOWED. WRITE SOLUTIONS ON THE SPACE PROVIDED.

Question	Mark
1	/10
2	/10
3	/10
4	/10
5	/10
TOTAL	/50

1.[10 points] Let V be a finite dimensional vector space over $\mathbb{C}$ with a scalar product $(\cdot, \cdot)$. Let $A : V \to V$ be a linear transformation. Define the notions of operator norm, Hilbert-Schmidt norm and spectral radius of A.

2.[10 points] Let V be the vector of functions in variable x spanned by $\{1, x, \sin(x), \cos(x)\}$. Let D be the differentiation operator with respect to x. Find the eigenvalues and eigenspaces of D. Find the characteristic polynomial and minimal polynomial of D.

3.[10 points] Let A be $n \times n$ complex matrix such that $||A|| = ||A^{-1}|| = 1$ (where $|| \cdot ||$ denotes the operator norm). Show that A is unitary.

4.[10 points] Suppose A is an $n \times n$ matrix such that $A^3 = I$. Prove that A is diagonalizable.

5.[10 points] Suppose A and B are positive definite matrices that commute i.e. $AB = BA$. Is it true that if $A > B$ then $A^2 > B^2$?