MATH 1050 (Combinatorics)
Final exam information sheet

Kiumars Kaveh

December 13, 2015

Topics discussed in the course (from Keller-Trotter):

- Section 2: Binomial Coefficients
- Section 3: Induction
- Section 4: Combinatorial Basics
- Section 5: Graph Theory
- Section 7: Inclusion-Exclusion (7.1-7.4)
- Section 8: Generating Functions (8.1-8.3)
- Section 9: Recurrence Equations (only 9.6)
- Section 11: Applying Probability to Combinatorics (only 11.0)
- Section 12: Graph Algorithms
- Section 13: Network Flows
- Extra topics: (1) Linear programming and (2) matching theory (see the links to the notes in the course webpage).

Some info

- The final is 1 hour and 50 minutes long.
- There will be $5-6$ questions (each possibly broken into few parts).
- Make sure you know how to do all the homework problems.
- One question asks to state some definitions and theorems (discussed in class) without proof.
- The exam is accumulative but there is more emphasis on the second half of the course.

Some definitions and concepts (you need to know)

- Binomial coefficients, multinomial coefficients, Catalan number, Ramsey number
- (simple) graph, directed graph, weighted graph, degree of a vertex, walk, path, cycle, bipartite graph, vertex coloring, chromatic number, planar graph, Eulerian graph, Hamiltonian graph, complete graph, clique, independent set, matching, vertex cover, tree, spanning tree, network graph, flow, cut, value of a flow, capacity of a cut

Some theorems (you need to know the statements)

- Binomial theorem (for positive or negative exponent), Inclusion-Exclusion Principle, Pigeonhole Principle, Ramsey's theorem.
- Euler's theorem about when a graph is Eulerian, Four color theorem, Cayley's theorem on number of labeled trees, Euler's formula for planar graphs, Kuratowski's theorem that a graph is planar iff it does not contain K_5 or $K_{3,3}$, Max-flow-min-cut theorem, König's theorem about maximum matching and minimum cover, Hall's theorem about perfect matchings.

Some things you should know:

- Solve a 2 dimensional linear programming and write its dual linear programming problem.
- Given a network and a flow on it find an augmenting path.
- Given a network and a cut find the capacity of the cut.
- Given a network, write the linear programming associated to the max flow problem and its dual min cut problem.
- Given a bipartite graph use Hall's theorem to show that it does or does not have a perfect matching.
- Number of surjections from $\{1,\ldots,n\}$ to $\{1,\ldots,m\}$ (using Inclusion-Exclusion).
- Number of derangements (using Inclusion-Exclusion).
- Given a bipatite graph draw the corresponding network graph (as in the proof of König's theorem).
- Write the (usual) generating function of a sequence.
- Find the (usual) generating function of a sequence defined recursively.
- The Catalan number C(n) and the formula for it i.e. $C(n) = \frac{1}{n+1}\binom{2n}{n}$ as well as a couple of problems that have Catalan numbers as answer (Exercise 33 in Section 2.9).

- You need to know the formula for the number of integer solutions of $n = x_1 + \cdots + x_k$ when $x_i > 0$, i.e. $\binom{n-1}{k-1}$, as well as the same problem but $x_i \geq 0$, i.e. $\binom{n+k-1}{k-1}$. You also need to know the generating functions of these, i.e. $F(x) = x^k/(1-x)^k$ and $F(x) = 1/(1-x)^k$ (all explained in class, see Section 8.2 in Keller-Trotter).