MATH 1250 (Abstract Algebra)
Final Information Sheet

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- The midterm is two hours long.
- Covers Sec. 18-23, 26, 27, 29, 31, 32, 48-51, 53, 54.
- There will be 8 questions (each possibly broken into few parts).
- The questions are similar to the homework problems. Make sure you can do all the homework problems.
- One question asks to state some definitions and theorems (discussed in class) without proof.
- The emphasis is on field extensions and Galois theory i.e. Sec. 29, 31, 48-51, 53, 54.
- Definitions: characteristic of a field, field extension, algebraic and transcendental elements, irreducible polynomial of an algebraic element, simple extension, finite extension, algebraically closed field, algebraic closure of a field, finite field, Frobenius automorphism, separable extension and perfect field, separable element, splitting field, Galois group or automorphism group $G(E/F)$, Galois extension (normal extension).
- Maybe asked to give the proof of one of the following:
  - Theorems 29.13 (irreducible polynomial of an algebraic element) and 29.18 (simple extension).
  - Theorem 31.3 (a finite extension is algebraic), Corollary 31.7 and Theorem 31.16 (an algebraically closed field has no proper algebraic extension).
  - Theorem 33.1 and Corollary 33.2 (A finite field has p^n elements).
  - Theorem 48.3 (Conjugation Isomorphism), 48.11 (Fixed Field).
- Need to know the statements of the following Theorems/Propositions/Lemmas:
  - Eisenstein Criterion (23.15), 23.20.

- Fundamental Homomorphism Theorem (26.17).
- 27.9, 27.11, 27.15, 27.19, 27.25.
- Kronecker's theorem (29.3), 29.12, 29.13, 29.18 (important).
- 30.23.
- 31.3, 31.4 (important), 31.7, 31.11, 31.15, 31.16, 31.17 (important), 31.21 (Zorn's lemma).
- 32.5, 32.6, 32.8, 32.9, 32.10, 32.11.
- 33.1, 33.2, 33.3, 33.5, 33.6, 33.10, 33.11, 33.12.
- 48.3, (important), 48.5, 48.6, 48.11, 48.14, 48.15, 48.19.
- 49.3 (important), 49.4, 49.5 (algebraic closure is unique up to isomorphism), 49.10.
- 50.3 (splitting field is preserved under isomorphism), 50.6, 50.7.
- 51.9, 51.13, 51.14, 51.15 (Primitive Element Theorem), 51.16.
- 53.2, 53.6 (important, Main Theorem of Galois Theory), 53.7 (not discussed in class but the case $F = Z_p$ was covered in a homework problem Sec. 51, Ex. 14.
- 54.2.

* There will be a problem about finite fields.
* There will be a problem to find all subfields of a Galois extension $F \subset E$ as well as all the subgroups of the Galois group, and put them in one-to-one correspondence according to the fundamental theorem of Galois theory. (e.g. $\mathbb{Q} \subset \mathbb{Q}(\sqrt{2}, \sqrt{3})$).
* There will be a problem about splitting fields.
* There will be a problem about primitive element theorem i.e. given a finite separable extension $F \subset E$ find α with $E = F(\alpha)$.
* There will be a problem about the cyclotomic extension i.e. $\mathbb{Q} \subset \mathbb{Q}(\zeta)$ where $\zeta = e^{2\pi i/n}$ is a primitive n-th root of unity.