MATH 1020, Homework 1

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Numbered problems are from the Kenneth Rosen's *Elementary number theory and its applications*, 6th Ed.
All problems have equal points.

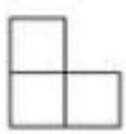
Due date: Wed. **September 12, 2018**

~~**Section 1.1: 4, 40**. If you like try Exercise 44 also which asks to prove e is irrational (no need to hand this one in).~~

Section 1.2: 10, 22 (alternatively you can guess a formula and then use induction to prove it).

Section 1.3: 4, 16, 26.

Problem: (Recursive algorithms) Consider a square of size 2^n where a 1×1 square is cut off from the lower left corner. Show that this shape (whose area is of course $2^{2n} - 1$) can be tiled with L-shaped tiles. An L-shaped tile is a 2×2 square from which a corner 1×1 square is removed (below). Hint: use induction on n.



(Bonus)

Problem: (An induction puzzle) There is a prisoner in a prison. He is sentenced to death (sob ...) and has been told that he will be killed on one day of the following week. He has been assured that the day will be a surprise to him, so he will not be anticipating the hangman on a particular day, so keeping his stress levels in check.

The prisoner happens to be a mathematician. He starts to argue that he will not be executed as follows: He thinks to himself, if I am still alive on Thursday, then clearly I shall be hanged on Friday, this would mean that I then know the day of my death, therefore I cannot be hanged on Friday. Now, if I am still alive on Wednesday, then clearly I shall be hanged on Thursday, since I have already ruled out Friday. The prisoner works back with this logic, finally concluding that he cannot after all be hanged, without already knowing which day it was.

His argument can be presented as a mathematical induction: let $P(n)$ be the statement that he will not be executed n days left to Saturday. By his argument above $P(1)$ is true i.e. he will not be executed on Friday. Now assume that $P(n-1), \ldots, P(1)$ are true (induction hypthesis). Then when there are n days left to the weekend, he knows that he will be executed on that day, which contradicts what he was promised. Thus $P(n)$ is also true. By induction he then concludes that he will not be executed at all.

Casually, resting on his laurels, sitting in his prison cell on Tuesday, the warden arrives to take him to be hanged, the prisoner was obviously surprised! What is wrong with his mathematical argument?