MATH 2370, Homework 1

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Problem 1: Prove that the vector space of all polynomials over $\mathbb{R}$ is *not* a finite dimensional.

Problem 2: Let V be the (infinite dimensional) vector space of all polynomials over $\mathbb{R}$. Let W be the subspace of polynomials of degree $\leq n$ where $n \geq 2$. Also fix distinct points $a, b \in \mathbb{R}$ and let U be the subspace of polynomials which vanish at both a and b. Determine the dimension of the subspace $U \cap W$. Give a precise proof for it using only material in Chapter 1.

Problem 3: Let U, V, W be subspace of a finite dimensional vector space. Suppose $W \cap (U + V) = (W \cap U) + (W \cap V)$. Prove that:

$$\begin{aligned}\dim(U + V + W) = \dim(U) + \dim(V) + \dim(W) - \dim(U \cap V) - \\ - \dim(U \cap W) - \dim(V \cap W) + \dim(U \cap V \cap W).\end{aligned}$$

Give an example to show that, without the assumption $W \cap (U + V) = (W \cap U) + (W \cap V)$, the above equality among dimensions may not hold.

Problem 4: Let V and W be finite dimensional vector spaces. Prove that $\dim(V \oplus W) = \dim(V) + \dim(W)$.