MATH 2371, Homework 1

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Problem 1: Show that if A is a complex Hermitian matrix then A has a square root, i.e. there is a complex matrix B such that $B^2 = A$. Show that if A is also positive then B can be chosen to be Hermitian and positive. Similarly, show that if A is a real symmetric matrix which is in addition positive, then there is a real symmetric positive matrix B such that $B^2 = A$.

Problem 2 Give an example of a (complex) 2×2 matrix (which is not self-adjoint) and does not have a square root. Prove your claim.

Problem 3: Let A be a normal matrix. Show that $||A|| = r(A)$ (as usual $||A||$ denote the operator norm and $r(A)$ is the spectral radius).

Problem 4: Consider the matrix

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

Compute the spectral radius of A^*A and use this to find the operator norm of A.

Problem 5: Let A be a $n \times n$ matrix (real or complex). Let e^A denote the exponential of A, that is:

$$e^A = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots .$$

Show that $\det(e^A) = e^{\mathrm{tr}(A)}$. Hint: Use Theorem 4 in Chapter 9 of Lax.