MATH 2371, Homework 2

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Problem 1: Compute the exponential e^A of the following matrices:

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Hint: either diagonalize the matrix or write the matrix as the sum of a diagonalizable matrix and a nilpotent matrix that commute, or any other method that you like!

Problem 2: Find a solution $x(t) = (x_1(t), x_2(t))$ for the differential equation system:

$$x_1'(t) = x_2(t)$$
$$x_2'(t) = x_1(t).$$

(Note: this is same as the differential equation $x_1''(t) = x_1(t)$.)

Problem 3:

(a) Give an example of a polynomial $p(t)$ and a square matrix valued function $A(t)$ such that $\frac{d}{dt}p(A)$ is not equal to $p'(A)\dot{A}$. So the usual chain rule formula may fail.

(b) But prove that *a trace of the chain rule remains*, that is, we always have:

$$\frac{d}{dt}\mathrm{tr}(p(A)) = \mathrm{tr}(p'(A)\dot{A}).$$

Problem 4: Let $A(t)$ be a differentiable square matrix valued function with $A(0) = I$. Calculate $\frac{d^2}{dt^2}\det(A(t))$ at $t = 0$ in terms of $\dot{A}$ and $\ddot{A}$.

Bonus Problem: Suppose $e^A = A$ show that A is diagonalizable. Hint: write A as $D + N$ where D is diagonalizable, N is nilpotent and $DN = ND$.