

# MATH 2371, Homework 3

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**Due date: Friday February 12, 2016**

**Problem 1:** Let  $A$  be a self-adjoint matrix. Show that if  $I < A$  then  $I < A^2$ .

**Problem 2:** For two  $n \times n$  matrices  $A, B$ , their Hadamard product  $A \circ B$  is defined as:

$$(A \circ B)_{ij} = A_{ij}B_{ij}.$$

Now let  $A, B$  be two  $n \times n$  positive matrices. Prove that  $A \circ B$  is positive using the notion of Gram matrix.

**Problem 3:** Give example of two  $2 \times 2$  matrices  $A, B > 0$  such that  $AB + BA$  is not positive.

**Problem 4:** Let  $A, B$  be self-adjoint  $n \times n$  matrices with  $B > 0$ . Prove that the eigenvalues of  $AB$  are real (note that  $AB$  is not necessarily a self-adjoint matrix).