MATH 3650, Symplectic Geometry

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All manifolds, functions, forms and vector fields are assumed to be C^∞, unless otherwise stated.

Problem 1: Let G be a Lie group acting smoothly on a manifold M. (a) For $\xi \in \mathrm{Lie}(G)$ let ξ_M denote the generating vector field of ξ. Also for $x \in M$ let $\mathcal{O}_x = \{g \cdot x \mid g \in G\}$ denote the G-orbit of x. Prove that ξ_M is tangent to $\mathcal{O}_x$, that is, $\xi_M(x) \in T_x\mathcal{O}_x$. (b) Show that the map $\xi \mapsto \xi_M(x)$ gives a surjective map from $\mathrm{Lie}(G)$ to $T_x\mathcal{O}_x$. Moreover, if H is the G-stabilizer of x, then $\xi \mapsto \xi_M(x)$ induces an isomorphism of vector spaces $\mathrm{Lie}(G)/\mathrm{Lie}(H)$ and $T_x\mathcal{O}_x$.

From DaSilva's book:

- Page 137 (linear and angular momentum). Show that the moment maps for translation action ($\mathbb{R}^3$) and rotation action ($\mathrm{SO}(3)$) on $\mathbb{R}^6$ are the ones given in the example.
- Homework 17 (Coadjoint orbits) 3–5.
- Homework 19 (Examples of moment maps) 2–5.