MATH 2370, Homework 4

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Problem 1: Let G be a finite group acting linearly on a vector space V. As discussed in class consider the averaging operator $P : V \to V$ defined by:

$$P(v) = \frac{1}{|G|} \sum_{g \in G} g \cdot v.$$

(a) Show that the image of P is the set of all invariant vectors:

$$V^G = \{v \in G \mid g \cdot v = v, \forall g \in G\}.$$

(b) Show that P is a projection i.e. $P^2 = P$.

Problem 2: Find the matrix of the following linear transformations $T : \mathbb{R}^2 \to \mathbb{R}^2$ with respect to the standard basis.

(a) T is the reflection with respect to the line $y + x = 0$.

(b) T is the rotation with angle α counter-clockwise around the origin.

Use the matrix you obtained in (b) and matrix multiplication to give a very quick proof of the trigonometric equalities: $\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$ and $\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$.

Problem 3: Let A be an $n \times n$ real matrix. Show that $AD = DA$ for all $n \times n$ diagonal matrices D, if and only if A itself is diagonal.

Problem 4: Let A, B be square matrices. Suppose A, B and $A + B$ are all invertible. Show that $A^{-1} + B^{-1}$ is also invertible. (Note that A and B may not commute.). Hint: Write $A^{-1} + B^{-1} = (A^{-1}B + I)B^{-1}$.