MATH 2371, Homework 4

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Due date: Friday March 4, 2016

Problem 1: Let A, B be positive matrices. Show that:

$$\det(A+B) \geq \det(A) + \det(B).$$

Problem 2: Let A be an $n \times n$ non-negative matrix. Show that:

$$\det(A) \leq (\frac{\mathrm{tr}(A)}{n})^n.$$

Problem 3:

(a) Let A be an $n \times n$ matrix. Show that A is nilpotent if and only if $p_A(t) = t^n$.

(b) Let A be a 2×2 matrix. Show that A is nilpotent if and only if $\det(A) = \mathrm{tr}(A) = 0$.

Problem 4: Suppose A and B are 2×2 matrices such that $AB + BA = 0$. Show that $AB = 0$ or $\mathrm{tr}(A) = \mathrm{tr}(B) = 0$.

Problem 5: Show that $\det(A) \neq 0$ where:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 2^2 & 3^2 & 4^2 \end{bmatrix}.$$