MATH 2370, Homework 5

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Problem 1: Let A be an $n \times n$ upper triangular matrix. Show that $\det(A)$ is the product of the diagonal entries of A.

Problem 2: Let A, B, C be $n \times n$ matrices. Is it always true that $\mathrm{tr}(ABC) = \mathrm{tr}(CBA)$?

Problem 3: Let A be an $n \times n$ matrix. Consider the function p defined by:

$$p(t) = \det(A - tI),$$

where I is the identity matrix and t is a scalar. Show that $p(t)$ is a monic polynomial of degree n:

$$p(t) = t^n + a_{n-1}t^{n-1} + \cdots + a_0,$$

and $a_{n-1} = (-1)^{n-1}\mathrm{tr}(A)$ and $a_0 = \det(A)$.

Problem 4: Let A be an invertible matrix. Show that there exists a polynomial f (in one variable) such that $f(A) = A^{-1}$. Note that the polynomial f depends on the choice of A. (Hint: observe that the set $\{I, A, A^2, A^3, \ldots\}$ is linearly dependent because the space of $n \times n$ matrices is finite dimensional, then use this to show that there is a polynomial g with $g(A) = 0$.)