MATH 2371, Homework 5

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Problem 1: Let A be an $n \times n$ real symmetric matrix.

(a) Let $d > 0$ be a real number. Show that $\lambda_{min}(A + dI) > \lambda_{min}(A)$.

(b) More generally, let D be a real diagonal matrix with positive diagonal entries. Show that $\lambda_{min}(A + D) > \lambda_{min}(A)$.

Problem 2: Let A, B, C be $n \times n$ matrices such that A is invertible, $B = AB$ and $C = -AC$. Prove that $rank(B) + rank(C) \leq n$. (Hint: show that $R_B \cap R_C = \{0\}$, where R_B, R_C are the range of B and C respectively.)

Problem 3: (Existence of logarithm) Let A be a positive matrix. Prove that there is a self-adjoint matrix B such that $e^B = A$. Show that B is unique.

Problem 4: Show if A, B are 3×3 matrices such that A, $B \neq 0$ but $A^2 = B^2 = 0$ then A and B are similar matrices.

Problem 5: Let A be an $n \times n$ complex matrix. Prove that if $\lim_{k \to \infty} A^k = 0$ then $r(A) < 1$, where $r(A)$ is the spectral radius.