MATH 2371, Homework 6

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Problem 1: Suppose A and B are positive matrices such that: $||A+B|| = ||A|| + ||B||$. Show that A and B have a common eigenvector.

Problem 2:

(a) When is a diagonal matrix unitary?

(b) Is it true that a unitary matrix always has a square root which is unitary? That is, if U is a unitary matrix, does there exist a unitary matrix V such that $V^2 = U$? Prove or give a counterexample.

Problem 3: Suppose A is a complex $n \times n$ matrix such that it has a unique eigenvalue λ and the corresponding eigenspace E_λ is one dimensional. Show that A has a cyclic vector, that is, there is a vector v such that the set $\{v, Av, A^2v, A^3v, \ldots\}$ spans the whole vector space $\mathbb{C}^n$.

Problem 4: Suppose A and B are commuting nilpotent matrices.

(a) Prove that $A + B$ is nilpotent

(b) Prove that AB is nilpotent.

(c) Give a counterexample to (a) and (b) if the nilpotent matrices do not commute.

Problem 5: Find the singular values of the matrix A. Find the operator norm of A.

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}.$$