MATH 2370, Homework 7

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Problem 1: Prove that the set of generalized eigenvectors corresponding to an eigenvalue λ form a subspace.

Problem 2: Let A be the matrix:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Find the eigenvalues of A. Find the generalized eigenspaces of A. Find the minimal polynomial and characteristic polynomial of A.

Problem 3: Find a basis for $\mathbb{R}^3$ consisting of generalized eigenvectors of A. Find the Jordan canonical form of A.

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix}.$$

Problem 4: Let A be a 5×5 rank one matrix, find all possible Jordan canonical forms of A. The order of Jordan blocks should be ignored.

Problem 5: Prove that a matrix A is diagonalizable (i.e. similar to a diagonal matrix) if and only if there is a basis for the vector space consisting of eigenvectors of A.

Problem 6: Let A be a nilpotent $n \times n$ matrix, i.e. $A^m = 0$ for some $m > 0$. Show that one can find $m \leq n$ such that $A^m = 0$.