MATH 1250 (Abstract Algebra)
Midterm Information Sheet

Kiumars Kaveh

February 11, 2014

- The midterm is 50 minutes long.
- Covers Sec. 18, 19, 20, 21, 22, 23, 24 (just quaternions), 26, 27.
- There will be 5 questions (each possibly broken into few parts).
- The questions are similar to the homework problems. Please make sure you can do all the homework problems.
- One question asks to state some definitions and theorems (discussed in class) without proof.
  - Definitions: ring, field, integral domain, division ring, ring homomorphism, ring isomorphism, zero divisor, unit, characteristic of a ring, Frobenius homomorphism, irreducible element in a ring (in particular an irreducible polynomial), ring of polynomials $F[x]$ (with coefficients in a field F), quaternions (an important example of a division ring), kernel of a homomorphism, ideal, maximal ideal, prime ideal, principal ideal.
  - Theorems: Fermat's and Euler's theorems (Theorems 20.1 and 20.8), Division Algorithm for $F[x]$ (Theorem 23.1), Factor Theorem (Corollary 23.3), Eisenstein Criterion (Theorem 23.15), Uniqueness of factorization in $F[x]$ (Theorem 23.20), Fundamental Homomorphism Theorem (Theorems 26.16 and 26.17, important!), Kronecker's theorem on existence of roots of polynomials (Basic goal: Theorem 29.3).
- Some other concepts you need to know: field of quotients of an integral domain (also called the quotient field), quotient ring (also called factor ring) R/N, the complex roots of the polynomial $x^n - 1$ i.e. n-th roots of unity $e^{2\pi ik/n}$, primitive n-th roots of unity $e^{2\pi ik/n}$ where $(k,n) = 1$, n-th cyclotomic polynomial $\Phi_n(x) = \prod_{(k,n)=1}(x - e^{2\pi ik/n})$.
- Proof of one of the following theorems: Cancellation property in an integral domain, a finite integral domain is a field, P is a prime ideal if and only if R/P is an integral domain, M is a maximal ideal if and only

if R/M is a field (R is assumed commutative), a field has no non-trivial ideals, an ideal containing a unit is the whole ring (Theorem 27.5), polynomial $x^{p-1}+\cdots+x+1$ is irreducible over $\mathbb{Q}$ (using Eisenstein's criterion), every maximal ideal is prime.

- Some other theorems you need to know (without proof): divisors of zero in $\mathbb{Z}_n$ (Theorem 19.3), every field is an integral domain (Theorem 19.9), evaluation homomorphism for polynomials (Theorem 22.4), every polynomial of degree n has at most n roots (Corollary 23.5), a polynomial of degree 2 or 3 is reducible if and only if it has a root (Theorem 23.10), reducibility of polynomials over $\mathbb{Z}$ vs. $\mathbb{Q}$ (Theorem 23.11), rational roots of polynomials (Corollary 23.12), Euclid's lemma for polynomials (Theorem 23.18), properties of a homomorphism (Theorem 26.3), construction of quotient ring (Theorem 26.9), every ideal in $F[x]$ or $\mathbb{Z}$ is principal (Example 27.22 and Theorem 27.24), prime ideals in $F[x]$ (Theorem 27.25).
- There will also a question asking to give examples of rings/ideals etc. with certain properties.
- Some useful examples of rings: $\mathbb{Z}$, $\mathbb{Z}_n = \mathbb{Z}/n\mathbb{Z}$, $\mathbb{Z}[\sqrt{2}] = \{a + b\sqrt{2} \mid a, b \in \mathbb{Z}\}$, $\mathbb{Z}[i] = \{a + ib \mid a, b \in \mathbb{Z}\}$, ring $M_n(R)$ of $n \times n$ matrices with entries in a commutative ring R, $R[x]$ where R is a commutative ring e.g. $\mathbb{Z}_n[x]$, $F[x]$ where F is a field e.g. $\mathbb{Q}[x]$, $\mathbb{Z}_p[x]$, quaternions $\mathbb{H}$, quotient rings R/N e.g. $\mathbb{Q}[x]/\langle x^2+1\rangle$ where $\langle x^2+1\rangle$ denotes the (principal) ideal generated by $x^2 + 1$.