MATH 0430, Review topics for midterm 1

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- You are responsible to know the homework problems, some problems in the test will be from homeworks or very similar to them.
- There will be proof problems (like the ones in the homeworks).
- Definitions: Group, subgroup, isomorphism, abelian group, cyclic group, order of an element in a group, cyclic subgroup generated by an element, generating set, permutation, orbit of an element in a permutation, cycle, transposition, odd and even permutations, symmetric group and alternating group, symmetry group of a figure, coset, direct product, homomorphism, normal subgroup.
- Important examples of groups: $(\mathbb{Z}_n, +)$, $(\mathbb{Z}_n^*, \times)$, dihedral group, Klein group V, symmetric group S_n, alternating group A_n, group of invertible $n \times n$ matrices (denoted by $\mathrm{GL}(n, \mathbb{R})$).
- Skills:
  - Verifying whether a given set and operation is a group/subgroup.
  - Determining whether two groups are isomorphic, or whether a given map is an isomorphism.
  - Decomposing a permutation as a product of disjoint cycles, as well as a product of transpositions.
  - Finding cosets of a subgroup.
  - You should know structure of cyclic groups, e.g. which elements are generators and how many subgroups it has (Theorem 6.14).
  - Determining the symmetry group of a simple figure e.g. square, hexagon.
  - Determine/prove whether a given map between groups is a homomorphism.
  - Drawing (multiplication) table of a group of small size.
  - Drawing Cayley digraph for a set of generators of a group of small size.

- Writing the list of all non-isomorphic finite abelian groups of a given order n.

- You should know the content of theorems: 4.15-4.18, 5.14, 5.17, 6.1, 6.6, 6.10, 6.14, 6.16, 7.4, 8.5, 8. 16, 9.8, 9.12, 9.15, 9.20, 10.1, 10.10-10.12, 11.2, 11.9, 11.1, 13.12, 13.18, 13.15, 13.18.

- Statement of theorems:
  - Cayley's theorem
  - Fundamental theorem of finitely generated abelian groups.

- Proof of theorems:
  - Elementary properties of groups, i.e. Theorems 4.15, 4.16, 4.17, 4.18.
  - Every subgroup of a cyclic group is cyclic (Theorem 6.6).
  - Lagrange's theorem
  - $\mathbb{Z}_n \times \mathbb{Z}_m \cong \mathbb{Z}_{mn}$ if and only if $(n, m) = 1$.
  - Theorem 13.15 which states that if H is the kernel of a homomorphism ϕ then $\phi(x) = \phi(y)$ if and only if x, y lie in the same left coset of H (similarly right coset).