MATH 0430, Review topics for midterm 2

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- You are responsible to know the homework problems (after midterm 1), some problems in the test will be from homeworks or very similar to them.
- Format of midterm 2 is similar to midterm 1.
- Definitions (groups): homomorphism, kernel, normal subgroup, factor group, simple group, center of a group, commutator subgroup, group G acting on a set X, transitive group action, stabilizer (isotropy) subgroup G_x, orbit $G \cdot x$.
- Definitions (rings): ring, field, homomorphism and isomorphism of rings, unit element, divisor of zero, integral domain, field of quotients of an integral domain, ring of polynomials, irreducible polynomial, evaluation homomorphism.
- Important examples rings: $\mathbb{Z}$, $\mathbb{Z}_n$, $n \times n$ matrices, polynomials $F[x]$ where F is a field, all functions from $\mathbb{R}$ to $\mathbb{R}$, direct product of rings.
- Important examples of fields: $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$ (complex numbers), $\mathbb{Z}_p$, $F(x)$ field of rational functions (field of quotients of polynomials).
- Statement of theorems (no proof):
  - Properties of homomorphisms (Theorem 13.15 and Corollary 13.18).
  - Kernel of a homomorphism is a normal subgroup (Corollary 13.20).
  - Factor group (Theorem 14.4 and Corollary 14.5).
  - Fundamental Homomorphism Theorem (Theorem 14.9 and 14.11).
  - Equivalent conditions for a subgroup to be normal (Theorem 14.13).
  - Normal subgroups in direct product (Theorem 15.8).
  - A_n is a simple group if $n \geq 5$ (Theorem 15.15).
  - Group actions (Theorem 16.12 and 16.14), Oobrit-Stabilizer Theorem (Theorem 16.16), and Burnside's formula (Theorem 17.1).
  - Elementary properties of rings (Theorem 18.8, 19.5).
  - Divisors of zero in $\mathbb{Z}_n$, and $\mathbb{Z}_p$ is a field (Theorem 19.3 and Corollary 19.12).

  - Field of quotients (Lemma 21.2, Theorem 21.5).
  - Polynomials: Division Algorithm (Theorem 23.1), Factor Theorem (Corollary 23.3).

- Proof of theorems:
  - Fundamental Homomorphism Theorem (Theorem 14.11)
  - Orbit-Stabilizer Theorem (Theorem 16.16)
  - Factor Theorem (Corollary 23.2) (using Division Algorithm).