HW8 Solutions

Sec. 33

6. $F_{31}^* = F_{31}\setminus\{0\}$ is a cyclic group of order 30 (under multi.). Hence it has $\phi(30) = \phi(5\cdot3\cdot2) = 4\cdot2\cdot1 = 8$ generators.

Thus it has 8 primitive 15th roots of unity.

9. Because both given poly. are irr. over $\mathbb{Z}_2$ (have no roots) then $\mathbb{Z}(\alpha)$ & $\mathbb{Z}(\beta)$ are both ext. of degree 3 and thus are subfields of $\overline{\mathbb{Z}_2}$ containing $2^3=8$ elements. By Thm. 33.3 both of these fields are fixed fields of Fr^3 (= roots of x^8-x) & hence $\mathbb{Z}(\alpha)=\mathbb{Z}(\beta)$ (as subfields of $\overline{\mathbb{Z}_2}$).

11. Because $\alpha\in F$ we have $\mathbb{Z}_p(\alpha)\subset F$. But because α is a generator of the multi. group F^* we see that $\mathbb{Z}_p(\alpha)=F$. Since $|F|=p^n$ the degree of α over $\mathbb{Z}_p$ must be n.

Sec. 48

4. The Conjugates of $\sqrt{2}-\sqrt{3}$ over $\mathbb{Q}$ are:

$\sqrt{2}-\sqrt{3}$, $\sqrt{2}+\sqrt{3}$, $-\sqrt{2}-\sqrt{3}$, $-\sqrt{2}+\sqrt{3}$ as in Ex. 48.17.

12. $$(\tau_5\tau_3)\left(\frac{\sqrt{2}-3\sqrt{5}}{2\sqrt{3}-\sqrt{2}}\right)=\tau_5\left(\tau_3\left(\frac{\sqrt{2}-3\sqrt{5}}{2\sqrt{3}-\sqrt{2}}\right)\right)$$

$$=\tau_5\left(\frac{\sqrt{2}-3\sqrt{5}}{-2\sqrt{3}-\sqrt{2}}\right)=\frac{\sqrt{2}+3\sqrt{5}}{-2\sqrt{3}-\sqrt{2}}.$$