Department of Mathematics

University of Pittsburgh

MATH 2371

Practice test, Spring 2016

Instructor: Kiumars Kaveh

Last Name: **Student Number:**

First Name:

TIME ALLOWED: 50 MINUTES. TOTAL: 50
NO AIDS ALLOWED. WRITE SOLUTIONS ON THE SPACE PROVIDED.

Question	Mark
1	/10
2	/10
3	/10
4	/10
5	/10
TOTAL	/50

1.[10 points] Suppose A is an $n \times n$ self-adjoint matrix. Show that A is nonnegative if and only if there is an $n \times n$ matrix B such that $A = B^* B$.

2.[10 points] Let V be vector space over $\mathbb{C}$ with scalar product $\langle \cdot, \cdot \rangle$. Let U be a unitary matrix (with respect to this scalar product). Suppose W is a subspace of V which is invariant under U, i.e. for any $w \in W$ we have $U(w) \in W$. Is it true that the orthogonal complement $W^{\perp}$ is also invariant under U? Prove your claim or give a counter-example.

3.[10 points] Suppose A^2 and $A^3 + A$ are diagonalizable matrices. Prove that A is also diagonalizable.

4.[10 points] Suppose A is a complex 2×2 matrix such that $\det(A) = 1$. Prove that A is unitary if and only if $||A|| + ||A^{-1}|| = 2$. As usual $||\cdot||$ denotes the operator norm.

5.[10 points] Let N be an $n \times n$ normal matrix (over $\mathbb{C}$). Prove that if for some vector x we have $N^2 x = 0$ then $Nx = 0$.