Department of Mathematics

University of Pittsburgh

MATH 2371

Practice test 2, Spring 2016

Instructor: Kiumars Kaveh

Last Name: **Student Number:**

First Name:

TIME ALLOWED: 50 MINUTES. TOTAL: 30
NO AIDS ALLOWED. WRITE SOLUTIONS ON THE SPACE PROVIDED.

Question	Mark
1	/10
2	/10
3	/10
TOTAL	/30

1.[10 points] Let $||M||_{HS}$ denote the Hilbert-Schmidt norm of an $n \times n$ matrix (linear operator) M on $\mathbb{C}^n$ with respect to the standard complex scalar product.

(a) Let A be an $n \times n$ complex matrix and let U be an $n \times n$ unitary matrix. Show that $||A||_{HS} = ||UA||_{HS}$.

(b) Let A be an $n \times n$ self-adjoint matrix with eigenvalues $\lambda_1, \ldots, \lambda_n$. Show that $||A||_{HS} = \sqrt{\sum_{i=1}^{n} \lambda_i^2}$.

2.[10 points] Let A be an $n \times n$ real matrix. Prove or disprove the following: A is orthogonal if and only if for all $x \in \mathbb{R}^n$ we have $||Ax|| = ||x||$. (Here $|| * ||$ is the standard Euclidean norm.)

3.[10 points] Suppose A is a diagonalizable matrix and N is a nilpotent matrix such that $A + N = I$. Prove that $A = I$.