Physics Laboratory Introduction
Modified by Beta Keramati – Summer 2008

Graphs

The following is a brief description of how graphs should be plotted.

The graph scales should increase in steps of 1, 2, or 5 (or multiples of these numbers by ten raised to some power) not 3, 4, 6, 7, 8 or 9, so that interpolation may easily be accomplished. Use decimal, not fractional notation. Scales for the graph must be chosen so that the graph at least halfway fills the paper in both directions.

The same scale need not be chosen for both axis and it is not always necessary that the intersection of the two axes be the zero value for either scale.

Each axis should be clearly labelled with the name of the quantity being plotted, its symbol and the units being used.
The graph should be given a descriptive title.

On some graphs a straight line or smooth curve should be drawn that best fits the points plotted. It may not pass through all of them; the extent to which it does is usually an indication of the reliability of the results.

If there is an "accepted form" that the graph is expected to take, this should be plotted instead of the "best fit line". The accepted form should be labelled as such and its equation should be stated or referenced, on the graph.

Uncertainties

One of the major purposes of doing Physics laboratories is to gain skill in realistically analyzing data and results obtained from the data.

The analysis of your data and results will be included in your report.
Although a detailed statistical analysis will not usually be necessary, a reasonable estimate of the uncertainty in any "final quantity" calculated is expected.

When we make a measurement the last digit of the number representing the measurement is usually estimated. The amount that we are uncertain by, in the quantity being measured, is called the "absolute uncertainty" in that quantity. The absolute uncertainty will be expressed as a ± value and will have the same units as the measured quantity.

Often it is more useful to express the size of the uncertainty relative to the size of the measurement.
This is called the "relative uncertainty" and may be found from:

$$\text{relative uncertainty in } A = \pm\, a/A$$

where A is the quantity being measured and ± a is the absolute uncertainty in A

Usually we multiply the relative uncertainty by 100% to find the “percent uncertainty” in the measured quantity. This provides a convenient way of expressing how precise a measurement is, as may be seen from the following example:

If the distance from Pittsburgh to Altoona is measured with the same absolute uncertainty (say ± 10 miles) as the distance from Pittsburgh to London, then the percent uncertainty will be less in the larger measurement.

Pittsburgh to London is 3300 miles.

$$^{\pm\,10\text{ miles}}/_{3300\text{ miles}} \times 100\% = \pm\,0.3\%$$

Pittsburgh to Altoona is 85 miles.

$$^{\pm\,10\text{ miles}}/_{85\text{ miles}} \times 100\% = \pm\,11.8\%$$

Causes of uncertainties

The uncertainty in a measurement may come from any of a variety of causes. Some will be discussed, first however it must be realized that mistakes are not counted as justified causes of experimental uncertainty. Pushing the wrong key on a calculator, using an instrument incorrectly or making a mathematical error can be avoided or at least corrected. True uncertainties are usually beyond our powers to do much about and the best that we may hope for is to obtain a reasonable estimate of how large they are. They represent the maximum amount by which we think the values may be in error.

When using a measuring instrument, if no other guide is available by which to judge the size of the uncertainty, then it is acceptable although usually somewhat pessimistic, to take the size of the smallest scale division as equal to the total absolute uncertainty.

For example, a reading taken from a thermometer calibrated in whole degrees might be quoted as ± 0.5 C°. Note the total uncertainty is twice the value quoted after the ± sign.

Often the uncertainty is due to equipment other than the measuring instruments. For instance, when using an air track, a glider's position may be recorded by sparks that jump from a wire on the glider to the frame of the track, leaving a series of marks on a strip of wax paper that they pass through. This is illustrated in Figure 1.

S* = distance between the marks on the paper.
S = distance moved by the glider between sparks.

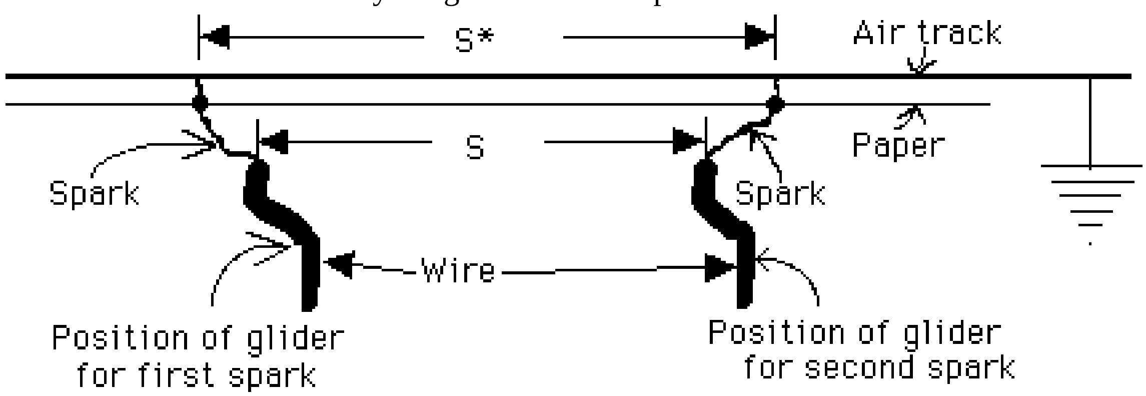

Figure 1. Spark timer in use with a glider and air track.

It can be seen that regardless of how well the distance between the spark marks on the paper, S*, is measured, this will not yield a perfectly accurate measurement of how far the glider moved, S, due to the non-consistent path of the spark.
For this length measurement it would be reasonable to estimate the uncertainty to be ± 2 mm as it is difficult to keep the wire any closer than a millimeter from the paper and the spark could jump as much sideways perhaps as forward, and we use two such marks. Taking into account the size of the marks themselves and the ruler used to make the measurement may cause us to quote the uncertainty to be as large as ± 3 mm.

The causes of uncertainty in any particular experiment will be discussed when it is performed in the laboratory.

Often the quantity that we are trying to evaluate is not measured directly, but is found from combining several other measurements that it is some function of.
Hence it is important to know how to combine uncertainties, so as to evaluate their net effect on some final result. This is discussed below.

Adding and Subtracting numbers with uncertainties

If the two quantities in the formula are either to be added or subtracted, then we add their absolute uncertainties. The reason for this may be seen from the following example.
Suppose a length C is measured in two sections, A and B, with each measurement being accurate to the extent quoted.

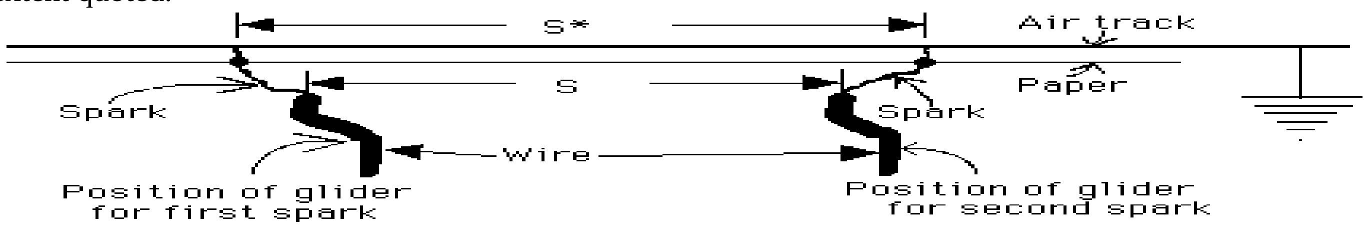

Figure 2. Quantities being added.

$$\text{Then } C \pm c = (A \pm a) + (B \pm b)$$

and as $C = A + B$, it follows that

$$\pm c = \pm (a + b)$$

If more than two terms are added then the process is repeated.
You should confirm for yourself that the same thing should be done if the quantities are subtracted.

$$\text{In general if} \quad X = (A + B - C + D - .....)$$

$$\text{then} \quad \pm x = \pm (a + b + c + d + ...)$$

Multiplying and dividing numbers with uncertainties

If the two quantities are multiplied or divided, then the relative or more usually the percent uncertainties are added.
For example, if we wish to find the area A, of the rectangle with a width of $X \pm x$, and a length of $Y \pm y$, we might multiply the length by the width.

$$(A \pm a) = (X \pm x)(Y \pm y)$$

$$(A \pm a) = XY \pm Xy \pm Yx \pm xy$$

now $A = XY$ and if we take xy to be small enough to be considered negligible, then:

$$\pm a = \pm Xy \pm Yx$$

therefore $(\pm {}^{a}/_{A}) = {}^{(\pm Xy \pm Yx)}/_{(XY)}$, which simplifies to

$$\pm \, ^{a}/_{A} = \pm \, (^{y}/_{Y} + \, ^{x}/_{X}) \text{ or}$$

$$(\pm \, ^{a}/_{A} \text{ x}100\%) = \pm \, \{(^{y}/_{Y} \text{ x } 100\%) + (^{x}/_{X} \text{ x } 100\%)\}$$

i.e. the % unc. A is equal to the % unc. in Y plus the % unc. in X

<u>Mixed operations</u>

If both addition or subtraction and multiplication or division occur in a formula, work from inside the brackets outward dealing first with the addition/subtraction parts and then with the multiplication/division parts, each time taking a pair of terms or more and reducing them to a single term. This is then repeated until the desired result is obtained.
For example suppose

$$R = \, ^{\{(A - B)C + D\}}/_{(E + F)} \qquad \text{Equation 1.}$$

and the quantities have been evaluated as follows:

A = 20.0 ± 0.1	B = 10.0 ± 0.2
C = 12.0 ± 0.1	D = 30.0 ± 0.5
E = 15.0 ± 0.2	F = 10.0 ± 0.3

Then $R = \, ^{\{(20 - 10)12 + 30\}}/_{(15 + 10)} = 6$

and r may be found from:

$$R \pm r = \, ^{\{(A \pm a - B \pm b)(C \pm c) + (D \pm d)\}}/_{(E \pm e + F \pm f)} \qquad \text{Equation 2.}$$

$$R \pm r = \, ^{\{(20.0 \pm 0.1 - 10.0 \pm 0.2)(12.0 \pm 0.1) + (30.0 \pm 0.5)\}}/_{(15.0 \pm 0.2 + 10.0 \pm 0.3)} \qquad \text{Equ. 3.}$$

The uncertainty in the term (A - B) is ± (a + b)

$$\text{i.e. } (A - B) = 10.0 \pm 0.3$$

Similarly the value of (E + F) is 25.0 ± 0.5

Substitution into Equation 3. yields:

$$R \pm r = \, ^{\{(10.0 \pm 0.3)(12.0 \pm 0.1) + (30.0 \pm 0.5)\}}/_{(25.0 \pm 0.5)} \qquad \text{Equation 4.}$$

The uncertainty in the term {(10.0 ± 0.3)(12.0 ± 0.1)} must be next evaluated. As the terms are multiplied together the absolute uncertainties must be converted to percent uncertainties before they are added.

$$(10.0 \pm 0.3) = 10.0 \pm (^{0.3}/_{10} \text{ x } 100\%) = 10.0 \pm 3\%$$

$$(12.0 \pm 0.1) = 12.0 \pm (^{0.1}/_{12} \text{ x } 100\%) = 12.0 \pm 0.83\%$$

$$\text{and } (10.0 \pm 3\%)(12.0 \pm 0.83\%) = 120 \pm 3.83\%$$

as 3.83% of 120 is 5 we may write Equation 4. as

$$R \pm r = {}^{\{(120 \pm 5) + (30.0 \pm 0.5)\}}/_{(25.0 \pm 0.5)}$$

$$R \pm r = {}^{(150 \pm 6)}/_{(25.0 \pm 0.5)}$$

Now 150 ± 6 = 150 ± 4% and 25 ± 0.5 = 25 ± 2%, therefore

$$R \pm r = ({}^{150}/_{25.0}) \pm (4\% + 2\%)$$

$$R \pm r = 6.0 \pm 6\%$$

6% of 6.0 is 0.4 and so the final answer is:

$$\underline{R = 6.0 \pm 6\%} \quad (\text{or } R = 6.0 \pm 0.4)$$

Uncertainties in trig functions

The uncertainty in a trigonometric function may be found if the uncertainty in the angle is known. From this the maximum and minimum values of the trigonometric function may be found and from these the most reasonable value and its associated uncertainty.
For example, sin(75° ± 5°) may take on values from sin 70° = 0.9397 to
sin 80° = 0.9848

The average of these two values is 0.9623 (cf. sin 75° = 0.9659)
Half the difference between them is 0.0226 = 2.34% of 0.9623
(cf. 5° is 6.67% of 75°)
Hence sin(75° ± 5°) = 0.9623 ± 2.3% (<u>not</u> 0.9659 ± 6.7%)

Plotting graphs with numbers that have uncertainties

If graphical analysis is used it may be necessary to find the uncertainty in the slope and/or the intercept of the line drawn. This may be done by drawing lines on the graph representing the maximum and minimum slopes that still appear to fit the data points plotted and lines showing where the maximum and minimum intercepts may be.

This method is not precise however it is quick and visual. The process is illustrated on the following diagram.

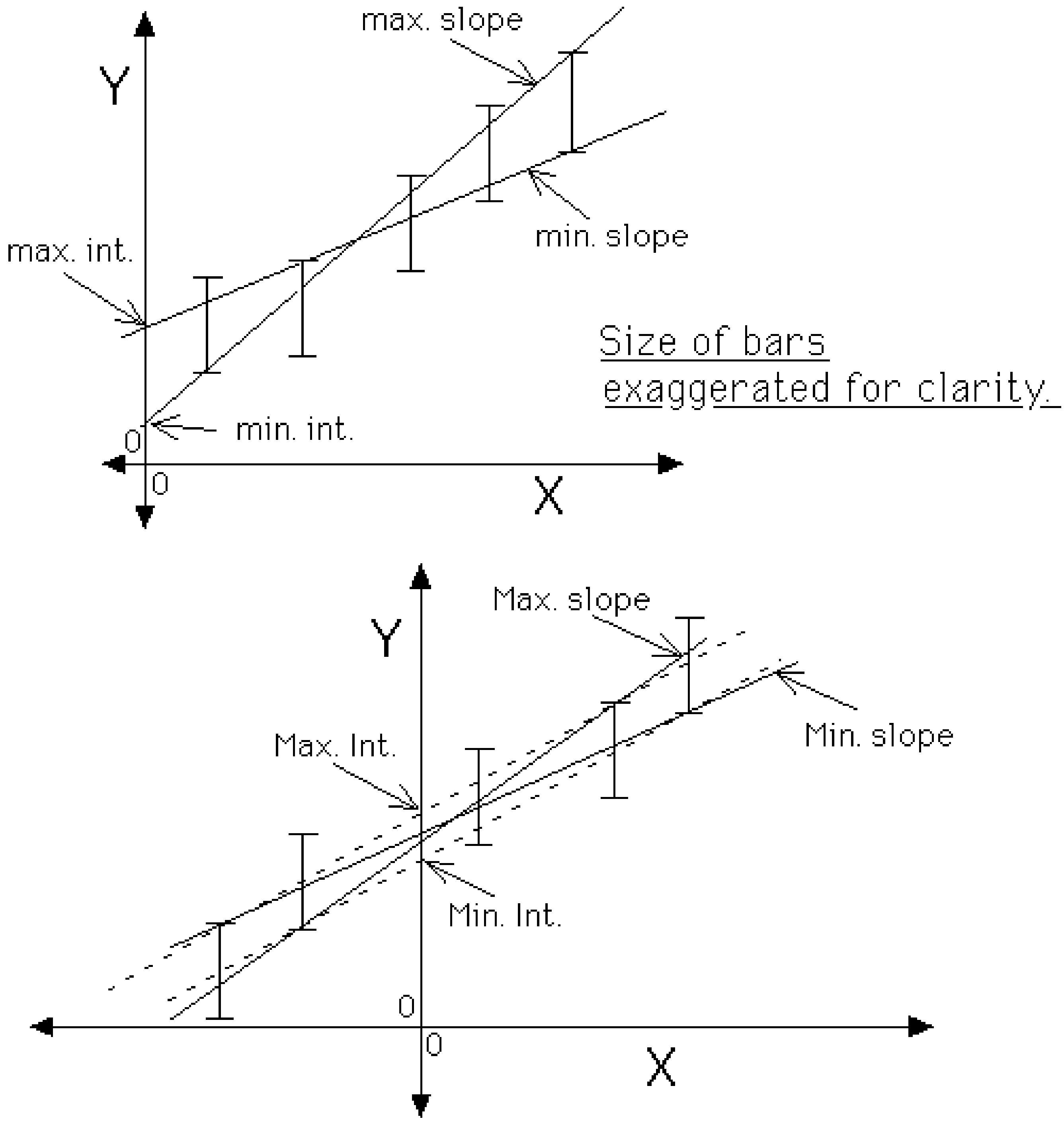

Figure 4. Lines to graphically evaluate uncertainty.

Uncertainties of multiple measurements

If a reading is repeated several times, but not enough to form the basis for a reliable statistical analysis, then usually we take the average value of the readings to represent the most probable size for the “true value” of the quantity being measured and the range of the readings as an estimate of the total absolute uncertainty in this average size.
For example, suppose that we measure the distance, S, a projectile travels when launched from a gun several times under supposedly the same conditions, and get:

Range of projectile, S, in meters

9.3	9.5	9.7
9.5	9.8	9.7
9.6	9.5	9.2
9.6	9.4	9.4

Then we would say that S =(9.5 ± 0.3)m as the average value is 9.5m and the highest and lowest values are 9.8m and 9.2m respectively.

Significant Figures

Every measurement has a degree of uncertainty associated with it. The uncertainty derives from the measuring device and from the skill of the person doing the measuring.

Let's use volume measurement as an example. Say you are in a chemistry lab and need 7 mL of water. You could take an unmarked coffee cup and add water until you think you have about 7 milliliters. In this case, the majority of the measurement error is associated with the skill of the person doing the measuring. You could use a beaker, marked in 5 mL increments. With the beaker, you could easily obtain a volume between 5 and 10 mL, probably close to 7 mL, give or take 1 mL. If you used a pipette marked to with 0.1 mL, you could get a volume between 6.99 and 7.01 mL pretty reliably. It would be untrue to report that you measured 7.000 mL using any of these devices, because you didn't measure the volume to the nearest microliter. You would report your measurement using significant figures. These include all of the digits you know for certain plus the last digit, which contains some uncertainty.

Significant Figure Rules

- Non-zero digits are always significant.
- All zeros between other significant digits are significant.
- The number of significant figures is determined starting with the leftmost non-zero digit. The leftmost non-zero digit is sometimes called the *most significant digit* or the *most significant figure*. For example, in the number 0.004205 the '4' is the most significant figure. The lefthand '0's are not significant. The zero between the '2' and the '5' is significant.
- The rightmost digit of a decimal number is the *least significant digit* or *least significant figure*. Another way to look at the least significant figure is to consider it to be the rightmost digit when the number is written in scientific notation. Least significant figures are still significant! In the

number 0.004205 (which may be written as 4.205×10^{-3}), the '5' is the least significant figure. In the number 43.120 (which may be written as 4.3210×10^{1}), the '0' is the least significant figure.

- If no decimal point is present, the rightmost non-zero digit is the least significant figure. In the number 5800, the least significant figure is '8'.

Significant Figures in Calculations

Measured quantities are often used in calculations. The precision of the calculation is limited by the precision of the measurements on which it is based.

- **Addition and Subtraction**
  When measured quantities are used in addition or subtraction, the uncertainty is determined by the absolute uncertainty in the least precise measurement (not by the number of significant figures). Sometimes this is considered to be the number of digits after the decimal point.

  Example
  32.01 m
  5.325 m
  12 m

  Added together, you will get 49.335 m, but the sum should be reported as '49' meters.

- **Multiplication and Division**
  When experimental quantities are mutiplied or divided, the number of significant figures in the result is the same as that in the quantity with the smallest number of significant figures. If, for example, a density calculation is made in which 25.624 grams is divided by 25 mL, the density should be reported as 1.0 g/mL, not as 1.0000 g/mL or 1.000 g/mL.

Losing Significant Figures

Sometimes significant figures are 'lost' while performing calculations. For example, if you find the mass of a beaker to be 53.110 g, add water to the beaker and find the mass of the beaker plus water to be 53.987 g, the mass of the water is 53.987-53.110 g = 0.877 g
The final value only has three significant figures, even though each mass measurement contained 5 significant figures.

Rounding and Truncating Numbers

There are different methods which may be used to round numbers. The usual method is to round numbers with digits less than '5' down and numbers with digits greater than '5' up (some people round exactly '5' up and some round it down).

Example:
If you are subtracting 7.799 g - 6.25 g your calculation would yield 1.549 g. This number would be rounded to 1.55 g, because the digit '9' is greater than '5'.

In some instances numbers are truncated, or cut short, rather than rounded to obtain appropriate significant figures. In the example above, 1.549 g could have been truncated to 1.54 g.

Exact Numbers

Sometimes numbers used in a calculation are exact rather than approximate. This is true when using defined quantities, including many conversion factors, and when using pure numbers. Pure or defined numbers do not affect the accuracy of a calculation. You may think of them as having an infinite number of significant figures. Pure numbers are easy to spot, because they have no units. Defined values or conversion factors, like measured values, may have units. Practice identifying them!

Example:
You want to calculate the average height of three plants and measure the following heights: 30.1 cm, 25.2 cm, 31.3 cm; with an average height of (30.1 + 25.2 + 31.3)/3 = 86.6/3 = 28.87 = 28.9 cm. There are three significant figures in the heights; even though you are dividing the sum by a single digit, the three significant figures should be retained in the calculation.

Accuracy and Precision

Accuracy and precision are two separate concepts. The classic illustration distinguishing the two is to consider a target or bulls-eye (we'll use arrows in this example). Arrows surrounding the bulls-eye indicate a high degree of accuracy; arrows very near to each other (possibly nowhere near the bulls-eye) indicate a high degree of precision. To be accurate an arrow must be near the target; to be precise successive arrows must be near each other. Consistently hitting the very center of the bulls-eye indicates both accuracy and precision.

Consider a digital scale. If you weigh the same empty beaker over and over and over again the scale will yield values with a high degree of precision (say 135.776 g, 135.775 g, 135.776 g). The actual mass of the beaker may be very different. Scales (and other instruments) need to be calibrated! Instruments typically provide very precise readings, but accuracy requires calibration. Thermometers are notoriously inaccurate, often requiring re-calibration several times over the lifetime of the instrument. Scales also require recalibration, especially if they are moved or mistreated.

Do you need more information and examples about significant figures? Visit this website for a tutorial on this subject.

Percent Error

If the purpose of the experiment is to find the value of some well established constant, such as when the size of "g" the acceleration due to gravity, is evaluated, then it is usual to compare the results with the "known, accepted or expected" value of the constant.

This is done by finding the "percent error" in the measured value.

$$\% \text{ error in } A = \{(A - A^*)/A^*\} \times 100\%$$

Where A = experimentally determined value and
A^* = known value.

It should be realized that the known, accepted or expected value is just what someone else measured using more precise equipment and techniques.

It follows that as the percent uncertainty represents the maximum amount that we suspect a quantity may be in error by, then the absolute value of the percent error should always be less than the percent uncertainty in the same quantity, if all has gone well.

Percent Difference

If we have two measured values for the same quantity, neither of which is the "known value", it is still often useful to express the difference between their sizes as a percentage.

$$\% \text{ difference between A and B} = \left\{ {}^{(A - B)}/_{(\text{average of A and B})} \right\} \times 100\%$$

where (average of A and B) = $^{(A + B)}/_{2}$

Error analysis in experiments

For each experiment you will be expected to include an analysis of the reliability of your results. This may most easily be accomplished by following the general procedure outlined below:

1. For each measured quantity, estimate its uncertainty.

2. Using the rules for calculating the uncertainties in derived quantities, calculate the percent uncertainty in your final answer.

3. Indicate the measurement or measurements that contributed most to the uncertainty in your final result and if possible, suggest simple changes in the experimental procedure that might lead to a significantly lower experimental uncertainty.

4. Calculate the percent error in your answer.

5. Compare the percent error with the percent uncertainty.

6. If the percent uncertainty is larger than the percent error, then you are probably justified in thinking that your result is valid.

7. If the percent error is appreciably larger than the percent uncertainty, then either you made a mistake, the equipment was not functioning correctly, or both. Check your measurements and calculations to see if a mistake has been made. If no such mistake is detected, the matter should be brought to the attention of the instructor and if possible, arrangements should be made to repeat the experiment. If this is not possible, you should at least attempt to identify the most probable cause for the error.

It is further expected that you will develop the ability to recognize which measurement in an experiment is limiting because of an inherent lack of precision. When this recognition is achieved, you will come to understand the futility of being extremely careful with some measurements or conditions when some other imprecise datum is limiting.

(I have a fossil that is 60,000,007 years old; I know it's that old because a Geologist told me that it was sixty million years old when I got it seven years ago.)

EXPERIMENT

Purpose: The purpose of this experiment is to gain practice in making good measurements, recording the uncertainties and including uncertainties in calculations to obtain the final result. The context is to determine the density of a piece of metal.

Apparatus:
A piece of metal
Triple beam balance
Ruler

Outline of Procedure:

- Measure and record the mass of the piece of metal and its associated uncertainty.
- Measure and record the dimensions of the piece of metal and their uncertainties.
- Calculate the volume, and then the density. Determine the uncertainty associated with density.
- Compare your result with the known density of the metal.
- Comment on your results.

See the Sample Lab Report posted on this website.