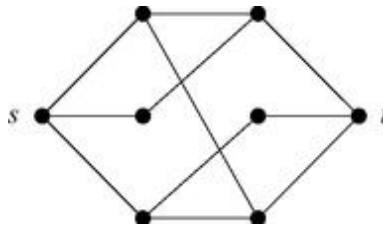


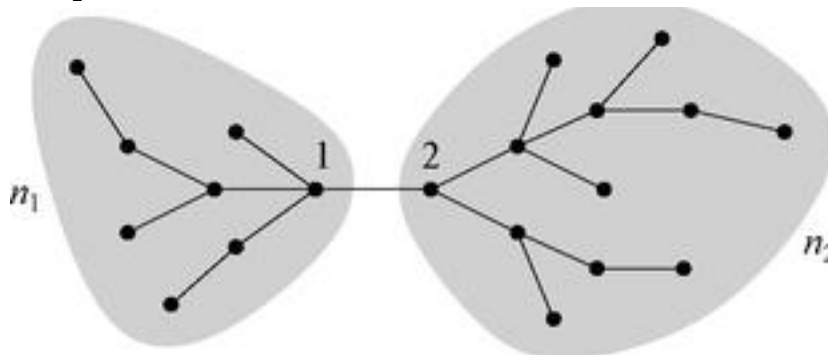
Assignment 1

- 1) Let \mathbf{A} be the adjacency matrix of an undirected network and $\mathbf{1}$ be the column vector whose elements are all 1. In terms of these quantities write expressions for:
 - a. The vector \mathbf{k} whose elements are the degrees k_i of the vertices
 - b. The number m of edges in the network
 - c. The matrix \mathbf{N} whose element N_{ij} is equal to the number of common neighbors of vertices i and j
 - d. The total number of triangles in the network
- 2) Consider a bipartite network, with its two types of vertex, and suppose that there are n_1 vertices of type 1 and n_2 vertices of type 2. Show that the mean degrees c_1 and c_2 of the two types are related by: $c_2 = \frac{n_1}{n_2} c_1$
- 3) What is the size k of the minimum vertex cut set between s and t in this network?



Prove your result by finding one possible cut set of size k and one possible set of k independent paths between set s and t . Why do these two actions constitute a proof that the minimum cut set has size k ?

- 4) Consider an undirected tree of n vertices. A particular edge in the tree joins vertices 1 and 2 and divides the tree in two disjoint regions of n_1 and n_2 vertices as shown below.



Show that the closeness centralities C_1 and C_2 of the two vertices,

are related by: $\frac{1}{C_1} + \frac{n_1}{n} = \frac{1}{C_2} + \frac{n_2}{n}$, where $C_i = \frac{n}{\sum_j d_{ij}}$

- 5) In a survey of couples in San Francisco, Catania *et al.* [1] recorded, among other things, the ethnicity of their interviewees and calculated the fraction of couples whose members were from each possible pairing of ethnic groups. The fractions were as follows:

		Women				Total
		Black	Hispanic	White	Other	
Men	Black	0.258	0.016	0.035	0.013	0.323
	Hispanic	0.012	0.157	0.058	0.019	0.247
	White	0.013	0.023	0.306	0.035	0.377
	Other	0.005	0.007	0.024	0.016	0.053
Total		0.289	0.204	0.423	0.084	

Assuming the couples interviewed to be a representative sample of the edges in the undirected network of relationships for the community studied, and treating the vertices as being of four types – black, Hispanic, white and other – calculate the modularity of the network with respect to ethnicity.

[1] J.A. Catania, T.J. Coates, S. Kegels and M.T. Fullilove, “**The population-based AMEN (AIDS in Multi-Ethnic Neighborhoods) study**”, in *Am. J. Public Health* **82**, 284-287 (1992).