Assignment 2

1) Suppose that a network has a degree distribution that follows the exponential form $p_k = Ce^{-\lambda k}$, where C and λ are constants.

a. Find C as a function of λ.
b. Calculate the fraction P of vertices that have degree k or greater.
c. Calculate the fraction W of ends of edges that are attached to vertices of degree k or greater.

2) Consider the following simple and rather unrealistic mathematical model of a network. Each of n vertices belongs to one of several groups. The m-th group has n_m vertices and each vertex in that group is connected to others in the group with independent probability $p_m = A(n_m - 1)^{-\beta}$, where A and β are constants, but not to any vertices in other groups. Thus, this network takes the form of a set of disjoint clusters or communities.

a. Calculate the expected degree $\langle k \rangle$ of a vertex within group m.
b. Calculate the expected value of $\bar{C}_m$ of the local clustering coefficient for vertices within group m.
c. Show that $\bar{C}_m \propto \langle k \rangle^{-\beta/(1-\beta)}$.
d. What value would β have to have for the expected value of the local clustering to fall off with increasing degree as $\langle k \rangle^{-3/4}$?

3) Consider the random graph G(n,p) with mean degree c. Also consider that S is the fraction of the network occupied by the giant component.

a. Calculate the probability that a vertex of degree k belongs to a small component as a function of S.
b. Show that the fraction of vertices in small components that have degree k is $\dfrac{e^{-c}c^k(1-S)^{k-1}}{k!}$.

4) The entropy H of a probability distribution p_i, $i \in \{1,2,\ldots,n\}$ is given by: $H(P) = -\sum_{i=1}^{n} p_i \log(p_i)$. Considering a network, its entropy is essentially the entropy of the corresponding degree distribution. Argue, quantitatively (preferred) or qualitatively, that if G_1 is a k-regular graph and G_2 is an Erdos-Renyi graph, that $H(G_1)<H(G_2)$.