Assignment 1

1) Let **A** be the adjacency matrix of an undirected network and **1** be the column vector whose elements are all 1. In terms of these quantities write expressions for:

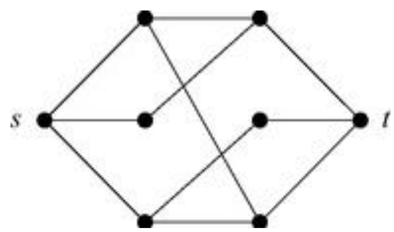
a. The vector **k** whose elements are the degrees k_i of the vertices

b. The number *m* of edges in the network

c. The matrix **N** whose element N_{ij} is equal to the number of common neighbors of vertices i and j

d. The total number of triangles in the network

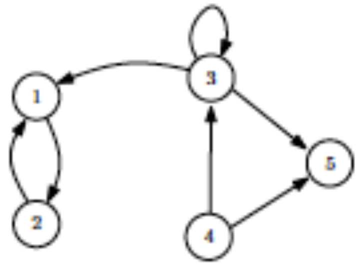
2) What is the size *k* of the minimum vertex cut set between *s* and *t* in this network ?



Prove your result by finding one possible cut set of size *k* and one possible set of *k* independent paths between set *s* and *t*. Why do these two actions constitute a proof that the minimum cut set has size *k*?

3) Show that the solution of $\pi^{*T} = \pi^{*T}G$, i.e., the page rank of the network described by the Google matrix G, is the same as the solution to the linear system: $(\mathbf{I}-\alpha\mathbf{H})^T\mathbf{x}=\mathbf{v}$, where **v** is the probability distribution vector used for the randomization and **H** is the original matrix describing the topology of the links.

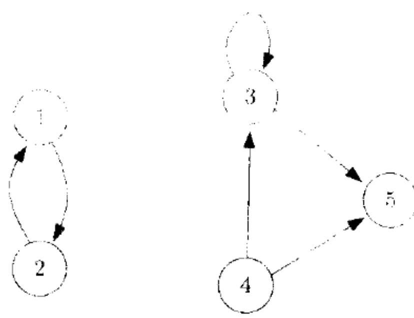
4) Compute the PageRank vector **x*** of the graph shown below for a=0.1, 0.3, 0.5 and 0.85. What do you observe? Use the iteration method and provide a snipet of the code.



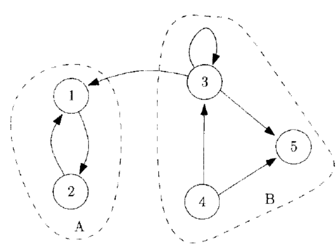
5) Set a = 0.85 and start with any normalized initial vector **x**[0].

a. Compute the PageRank vector $[x_A^* \; x_B^*]^T$ of the gaph in the following figure with: $\boldsymbol{H} = \begin{bmatrix} 1 & 0 \\ 1/3 & 2/3 \end{bmatrix}$. Note the uneven splitting of link weights from node B. This will be useful later in the problem.

b. Compute the PageRank vectors $[x_1^* \; x_2^*]^T$ and $[x_3^* \; x_4^* \; x_5^*]^T$ of the two graphs shown below.



c. If we divide the graph of problem 4 into two blocks as shown in the following figure, we can approximate $\mathbf{x}^*$ in problem 4 by: $\widetilde{\boldsymbol{x}^*} = [x_A^*[x_1^* \; x_2^*] \;\; x_B^*[x_3^* \; x_4^* \; x_5^*]]^T$. Compute this vector. Explain the advantage, in terms of computational load, of using the approximation instead of directly computing $\mathbf{x}^*$.



Use the iteration method and provide a snipet of the code.