Assignment 2a

1) Suppose that a network has a degree distribution that follows the exponential form $p_k = Ce^{-\lambda k}$, where C and λ are constants (20pts).

a. Find C as a function of λ.
b. Calculate the fraction P of vertices that have degree k or greater.
c. Calculate the fraction W of ends of edges that are attached to vertices of degree k or greater.

2) Consider the following simple and rather unrealistic mathematical model of a network. Each of n vertices belongs to one of several groups. The m-th group has n_m vertices and each vertex in that group is connected to others in the group with independent probability $p_m = A(n_m - 1)^{-\beta}$, where A and β are constants, but not to any vertices in other groups. Thus, this network takes the form of a set of disjoint clusters or communities (20 pts).

a. Calculate the expected degree $\langle k \rangle$ of a vertex within group m.
b. Calculate the expected value of $\bar{C}_m$ of the local clustering coefficient for vertices within group m.
c. Show that $\bar{C}_m \propto \langle k \rangle^{-\beta/(1-\beta)}$.
d. What value would β have to have for the expected value of the local clustering to fall off with increasing degree as $\langle k \rangle^{-3/4}$?

3) Consider a "line graph" consisting of n vertices in a line like this:

a. Show that if we divide the network into two parts by cutting any single edge, such that one part has r vertices and the other has n-r, the modularity, Eq. (7.76), takes the value: $Q = \dfrac{3 - 4n + 4rn - 4r^2}{2(n-1)^2}$
b. Show that when n is even, the optimal such division, in terms of modularity, is the division that splits the network exactly down the middle.

(20 pts)