Mean Field Approximation (MFA)

Instead of dealing with the full randomness of a network formation/growth process, and trying to calculate the exact degree distribution, we approximate it through the distribution of the **expected** degree. With Mean Field Approximation (MFA) we study the behavior of large complex systems by studying a simpler model. In the latter, we essentially "hide" the interactions between the many parts of the model and we abstract them to a single average effect.

Let us know see an example of MFA when trying to estimate the degree distribution for the BA model. Let us further assume that every node is labeled with the time that it was created, that is, node i is created at time i.

At time t there are c new edges generated. Node i will obtain each one of them with a probability proportional to its current degree k_i. Hence, the rate of increase for his degree k_i satisfied the following equation:

$$\left.\begin{aligned} \frac{dk_i(t)}{dt} &= c\frac{k_i}{2tc} \\ k_i(i) &= c \end{aligned}\right\} \Rightarrow k_i(t) = c\left(\frac{t}{i}\right)^{1/2}$$

Using the above equation we can see that the average time of birth for a node with degree k' is: $\tau = t\left(\frac{c}{k'}\right)^2$

Nodes that were born after this time are expected to have degree less than k'. Hence, if we want to find the fraction of nodes that have degree less than k' at time t (this is the CDF of the degree distribution) we simply have to calculate the fraction: $(t-\tau)/t$ (recall that the labels of the nodes correspond to the time they were created). Therefore,

$$\Pr(k < k') = 1 - \frac{c^2}{k'^2} \Rightarrow p(k) = \frac{2c^2}{k^3}$$