MIDTERM 1 : Math 1350 : Fall 2014
GOOD LUCK!!!

Problem 1. (10pts)
Consider the space curve:

$$\gamma(t) = \left(\frac{1}{2}(\sin t - \cos t), \frac{1}{2}(\cos t + \sin t), \frac{\sqrt{2}}{2} t \right)$$

Show that γ is parametrised by arc-length, and find the Frenet frame, torsion and curvature at the point $t = \pi/2$.

Problem 2. (20pts)
Let $\gamma : I \to \mathbb{R}^2$ be a regular parametrized curve on a closed interval I. Let r be a fixed nonzero number. We define the parallel curve to γ by $\gamma_r : I \to \mathbb{R}^2$, where:

$$\gamma_r(t) = \gamma(t) + r\vec{n}_s(t)$$

and $\vec{n}_s$ denotes, as usual, the signed normal vector to γ.
(1) Prove that γ_r is regular if and only if $\frac{1}{r} \notin [\kappa_{min}, \kappa_{max}]$, where:

$$\kappa_{min} = \min_{t \in I} \kappa_s(t), \quad \kappa_{max} = \max_{t \in I} \kappa_s(t).$$

(2) Now suppose that γ_r is regular and that γ is a closed simple curve, oriented anticlockwise. Show that the length of γ_r is related to the length of γ by:

$$l(\gamma_r) = l(\gamma) - 2\pi r.$$

Problem 3. (20pts)
(1) Find the tangent plane of a surface which is the graph of a smooth function $z = f(x, y)$ at the point $p_0 = (x_0, y_0, f(x_0, y_0))$. Your answer should depend on the function f and its derivatives at (x_0, y_0).
(2) Suppose a regular surface is given by the graph of:

$$z = xg(y/x), \quad x \neq 0,$$

where $g : \mathbb{R} \to \mathbb{R}$ is a smooth function. Show that its tangent plane at any point passes through the origin $(0, 0, 0)$.