MIDTERM 2 : Math 1350 : Fall 2014
GOOD LUCK!!!

Problem 1. (15pts)
Consider the helicoid S parametrised by:

$$\sigma(u,v) = (v\cos u, v\sin u, u) \qquad \forall (u,v) \in \mathbb{R}^2.$$

(1) Show that σ above is a regular parametrisation of a ruled surface.
(2) Compute the Gaussian curvature and the mean curvature of S.

Problem 2. (15pts)
(1) State the definition of a geodesic curve on a surface S and prove that it must have constant speed.
(2) Prove that every geodesic has zero geodesic curvature.
(3) Is the curve: $\gamma(t) = \sigma(\frac{t}{\sqrt{10}}, 3)$ a geodesic on the surface S in problem 1?

Problem 3. (20pts)
Let S be an oriented surface, whose unit normal vector at a point $p \in S$ we denote by $N(p)$. For every $\lambda \in \mathbb{R}$ such that $|\lambda| \ll 1$ is very small, define:

$$S^\lambda = \big\{p + \lambda N(p);\ p \in S\big\}.$$

(1) Prove that S^λ is an oriented surface and find its normal vector N^λ.
(2) Find the Weingarten map $\mathcal{W}^\lambda$ of S^λ in terms of the Weingarten map $\mathcal{W}$ of S.
(3) Prove that the principal curvatures of S^λ are given by:

$$\kappa_1^\lambda = \frac{\kappa_1}{1-\lambda\kappa_1}, \qquad \kappa_2^\lambda = \frac{\kappa_2}{1-\lambda\kappa_2},$$

where κ_1 and κ_2 are the principal curvatures of S.