MIDTERM 1 : Math 1700 : Spring 2014
GOOD LUCK!!!

Problem 1. (5+5 points)
Let (X, d) be a metric space.
(i) Show that if a ball in X of radius 7 is a subset of a ball of radius 3, then these balls must be the same.
(ii) Can a ball in X of radius 4, be a proper subset of a ball of radius 3?

Problem 2. (10+5 points)
Let $\{X_\alpha\}$ be a family of topological spaces. For each α, let A_α be a subset of X_α.
(i) Prove that: $\overline{\prod_\alpha A_\alpha} = \prod_\alpha \bar{A}_\alpha$, where the closure in the left hand side is taken with respect to the product topology.
(ii) Is the same true for the box topology?

Problem 3. (5+10 points)
Let X, Y be two topological spaces, and let $f : X \to Y$ be a function.
(i) Prove that if f is continuous, then for every convergent sequence $x_n \to x_0$ in X, the sequence $f(x_n)$ converges to $f(x_0)$ in Y.
(ii) Is the converse true?

Problem 4. (10 points)
Let X and Y be two topological spaces and let Y be Hausdorff. Given a function $f : X \to Y$, define G_f (called the graph of f) to be the subspace:

$$G_f = \{(x, f(x));\ x \in X\}$$

of $X \times Y$. Prove that if f is continuous, then G_f is closed.

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Bonus Problem. (15 points)
Prove that the cartesian product of an arbitrary family of connected topological spaces is connected in the product topology.