MIDTERM 2 : Math 1700 : Spring 2014
GOOD LUCK!!!

Problem 1. (10 points)
Let X be a compact topological space and for each $i \in \mathbb{N}$ let $F_i \subset X$ be a closed, nonempty subset of X. Assume further that the subsets F_i are nested, i.e.:

$$\forall i \quad F_{i+1} \subset F_i.$$

Using only the definition of compactness, prove that the set $\bigcap_{i=1}^{\infty} F_i$ is nonempty.

Problem 2. (10 points)
Let X and Y be two topological spaces. Show that $X \times Y$ (with the product topology) is separable if and only if X and Y are both separable.

Problem 3. (10 points)
Let X be a topological space which is completely regular (i.e. $T_{3\frac{1}{2}}$). Let $A, B \subset X$ be two closed, disjoint subsets, and assume that A is compact. Prove that there exists a continuous function $f : X \to [0,1]$ such that $f_{|A} = 0$ and $f_{|B} = 1$.

Problem 4. (10 points)
Show that if X is a locally compact Hausdorff space then its one-point compactification is also Hausdorff.

Problem 5. (10 points)
Let (X, d) be a metric space.
(i) If X is Lindelöf, show that X is second countable.
(ii) If X is separable, show that X is second countable.

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Bonus Problem. (20 points) (Only complete/almost-complete solution to this problem will be awarded points)
Let (X, d) be a metric space. Show that X is compact if and only if every continuous function $f : X \to \mathbb{R}$ is bounded.