MIDTERM 1, Math 3020, Spring 2012

(19 March 2012)

Problem 1. (20 points)
Prove that for any sufficiently large viscosity ν, there exists unique weak solution to the stationary Navier-Stokes equations in a 3d open bounded domain.

[The a-priori bound has to be proven in detail.]

Problem 2. (15 points)
State the Murat-Tartar div-curl lemma.

[All the notions have to be explained in detail, particularly the notions of convergences and compactness]

Problem 3. (15 points)
State the Friesecke-James-Muller rigidity theorem. Find the explicit formula for the optimal rotation. You may assume that $\fint \nabla u$ is close to $SO(n)$.