

**Introduction to Abstract Algebraic Systems MATH-430-1070 (11365),
Fall 2022**

Bonus Problem 1 (two weeks time, worth 15 points)

Consider the set: $G = \{e, e', i, i', j, j', k, k'\}$.

(i) Show that it is possible to define the binary operation in G with the properties: e is the unity elements, $(e')^2 = e$, $i^2 = (i')^2 = j^2 = (j')^2 = k^2 = (k')^2 = e'$, $ij = k$, $jk = i$, $ki = j$, $ji = k'$, $kj = i'$, $ik = j'$. Show that the above conditions uniquely determine the group (G, \cdot) . This group is called the *quaternion group*.

(ii) Find all subgroups of the quaternion group.