**Introduction to Abstract Algebraic Systems (MATH-430-1070 (11365), Fall 2022**

# Exercise Set 1

**1.** Determine whether the following sets and operations form a group or not (prove or disprove the group properties):
   (i) Invertible lower-triangular $n \times n$ matrices, with matrix multiplication.
   (ii) $\mathbb{Z}$ with product defined in: $a \cdot b = a + b + 2$.
   (iii) The interval $[0, 1)$ with product defined in:
$$a \cdot b = \begin{cases} a + b & \text{if } a + b < 1 \\ a + b - 1 & \text{if } a + b \geq 1. \end{cases}$$

**2.** Show that if every element of a group $G$ is its own inverse, then $G$ is abelian.

**3.** Determine whether the following subsets are subgroups of $(\mathbb{R}, +)$:

$$\mathbb{Q}, \quad \mathbb{Z}, \quad \mathbb{N}.$$

**4.** Determine whether the following subsets are subgroups of $(\mathbb{R}, +)$:

$$\{a + b\sqrt{2};\ a, b \in \mathbb{Z}\}, \quad \mathbb{R}_+.$$

**5.** Artin textbook problem 3.2, chapter 2.