Introduction to Abstract Algebraic Systems MATH-430-1070 (11365), Fall 2022

Exercise Set 2

1. Let H_1, H_2 be two subgroups of a group $(G, \cdot)$. Show that the following set:
$$H_1 H_2 = \{h_1 h_2; \; h_1 \in H_1, \; h_2 \in H_2\}$$
is also a subgroup if and only if $H_1 H_2 = H_2 H_1$.

2. Let $(G, \cdot)$ be a group and let $A \subset G$ be its nonempty subset. We define the *centralizer* of A as:
$$Z_G(A) = \{g \in G; \; \forall a \in A \; ga = ag\}.$$
Show that the centralizer is always a subgroup.

3. Prove or disprove the following statement: a given subgroup is the centralizer of any subset of its own centralizer.

4. Artin textbook problem 4.5, chapter 2.

5. Artin textbook problem 5.1, chapter 2.